CALIFORNIA INSTITUTE OF TECHNOLOGY

EARTHQUAKE ENGINEERING RESEARCH LABORATORY
Center for Research on the Prevention of Natural Disasters

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EERL 73-07

A Report on Research Conducted under Grants
from the National Science Foundation and
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ABSTRACT

The Millikan Library on the campus of the California Institute of Technology was strongly shaken during the San Fernando earthquake of February 9, 1971. The building was not damaged structurally, but the observed E-W response of the building showed a fundamental period of about 1.0 sec, significantly longer than the 0.66 sec observed in pre-earthquake vibration tests. In this study, the response of the fundamental mode was treated as that of a single-degree-of-freedom hysteretic structure, and four simple models, two stationary and two with changing properties, were examined to see if they could describe the observed response. It was found that an equivalent linear model and a bilinear hysteretic model both could match the response, provided their properties were changed during the earthquake. (Four changes were used). A linear model with constant properties and a stationary, bilinear hysteretic model did not give nearly as good agreement as the nonstationary models. The results indicated, in general, a degrading of the stiffness and energy dissipation capacity of the building, but it could not be determined whether the changes were sudden or gradual.

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INTRODUCTION

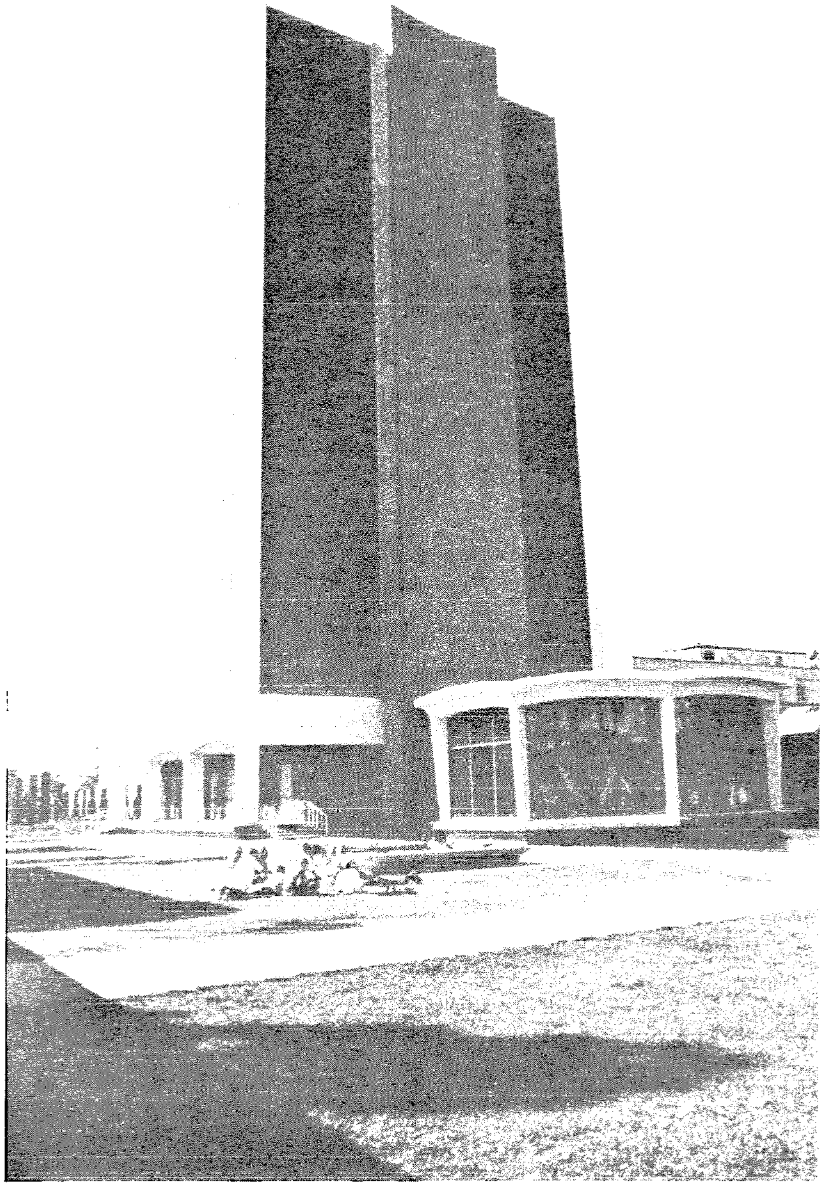
It is the intent of most modern approaches to earthquake-resistant design to produce a structure capable of responding to moderate shaking without damage, and capable of resisting the unlikely event of very strong shaking without seriously endangering the occupants. In the second case, however, structural damage and large deflections are permissible provided collapse is not imminent. To achieve this goal, it is necessary to understand the way buildings and other structures respond to deflections beyond the elastic limit, and much analytical and experimental work has been directed in recent years toward developing the required knowledge. In this effort, the development of analytical models for hysteretic behavior has been guided almost exclusively by static tests of structural elements and assemblages because it is not yet possible to excite full-scale structures significantly into the yielding range, and because the response of structures that have been heavily damaged under the action of strong earthquake motion has not yet been recorded. Thus, the desired full-scale, dynamic, confirmation of the approaches to the analysis of earthquake response of yielding structures have not yet been obtained.

In many studies of nonlinear response to earthquake motions or other dynamic forces, the yielding properties of structures have been modelled by the well-known elasto-plastic or bilinear force-deflection relations. References 1 and 2 are among the earliest works, and Reference 3 is one of the several studies presented at the Fifth World Conference on Earthquake Engineering which used these relations. In addition to these simple yielding relations, trilinear (4) and smoother but more complex models of yielding behavior (5) have also been used in studies of earthquake response. Some of the most recent work in this area includes the development of models for the deteriorating hysteresis evidenced by structures that are weakened by excursions beyond the elastic limit (6, 7).

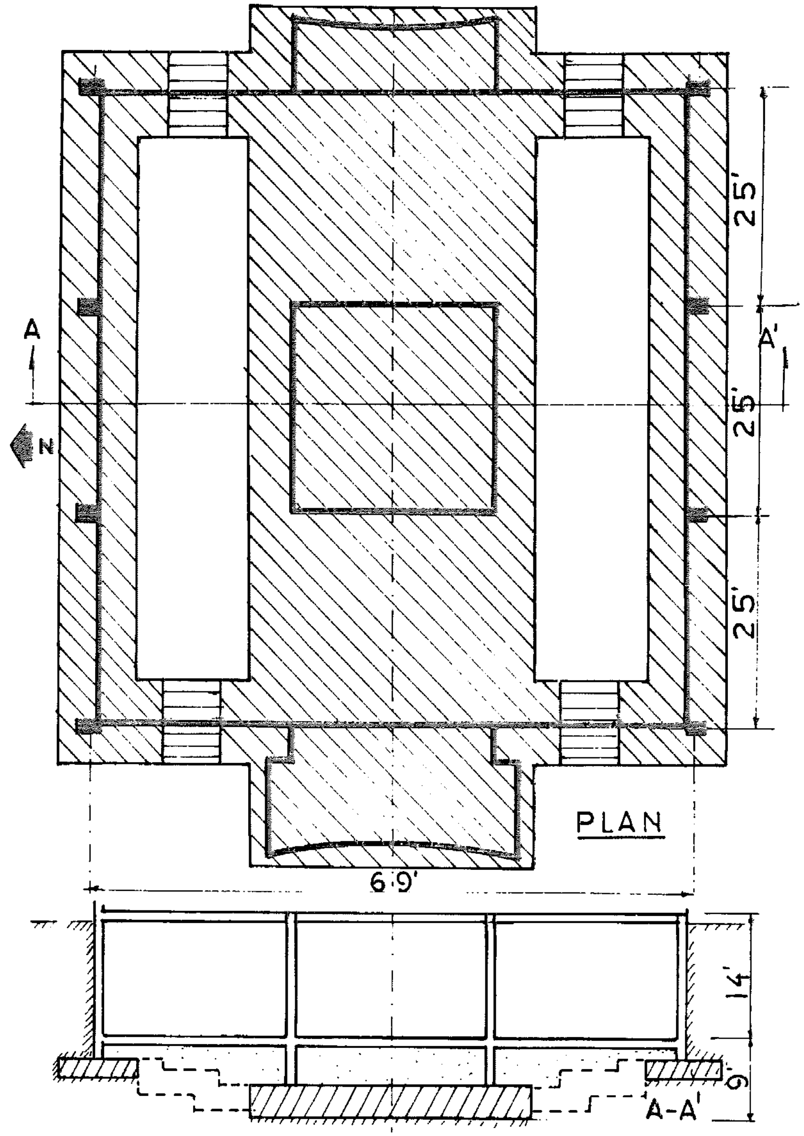
The occurrence of an earthquake can be viewed as a full-scale experiment and it is possible to learn much about the properties of structures from examination of their response to strong shaking (8). The largest collection of data of this type is from the recent San Fernando earthquake (9) in which responses of about 50 instrumented buildings in the Los Angeles area were obtained. None of the instrumented buildings was heavily damaged, but some did show evidence of nonlinear behavior in the form of lengthening of periods of the lower modes of vibration over those found from low-level vibration tests. A particular example of this occurred in the E-W response of the Millikan Library on the campus of the California Institute of Technology. The earthquake motion was measured at the basement and at the roof by two RFT-250 accelerographs which recorded the N-S, E-W and vertical components of the earthquake motion and building response. During the earthquake, the E-W motion at the roof reached a peak value of 340.5 cm/sec^2 (35%g), and clearly showed the fundamental period to be about one second, which is 50% greater than the value of 0.66 secs determined from forced vibration tests performed before the earthquake (10, 11)(the N-S response showed about a 20% reduction in the fundamental natural frequency. Visual examination of the E-W accelerogram and Fourier analysis of the record (12, 13) suggested that the library responded to the earthquake motion as a hysteretic structure to a degree that might make it a useful object of study.

The only observed effects in the building after the earthquake that might bear on the E-W response were small cracks at some floors in the interior plaster at the points of supports of the precast window wall panels. The exterior of the panels can be seen on the south face of the building in Figure 1. Because the building suffered no observable structural damage and because only the earthquake response at the top floor was available, it was not considered justified to make a detailed, nonlinear model of the structure of the library. It was decided instead to treat the response of the library in its fundamental mode as a single-degree-of-freedom hysteretic structure. The intent of this approach was to learn in a general way about the response of the building during the earthquake, and also to develop techniques of analysis that may be useful when the response of damaged structures is obtained in the future. In particular, we were interested in finding out if the response of the library in its fundamental mode could be satisfactorily described by one of the simpler models for hysteretic behavior.

As described in this report, several methods were tried in the attempt to find simple descriptions for the nonlinear response of the building during the earthquake. First, the measured motion of the fundamental mode of the structure was examined to see if the hysteretic relation could be determined from the measured earthquake response. It was found that, with some care, the dynamic force-deflection relation of the fundamental mode could be recovered. Another portion of this analysis was concerned with the nonstationary characteristics of the response in terms of the parameters of equivalent fundamental frequency and equivalent viscous damping. The changes of these variables during the response guided the selection of nonlinear models of the structure. The second major portion of the study was devoted to a comparison of the recorded response of the fundamental mode with that predicted by analyses using various hysteretic models. The models included a



(a)



(b)

FIGURE 1

Millikan Library Building at the California Institute of Technology
a) general view, and b) foundation and framing details.

stationary linear model with damping and frequency characteristics chosen to match the recorded response, a stationary bilinear model, and two non-stationary models. The nonstationary models were of two types, an equivalent linear model with damping and fundamental frequency that changed at selected times during the earthquake, and a nonstationary, bilinear hysteretic model whose properties also were changed during the response. The study closes with a discussion of the accuracy of the various methods proposed, and conclusions about their application to the response of Millikan Library.

MILLIKAN LIBRARY AND THE RECORDED EARTHQUAKE RESPONSE

Brief Description of Millikan Library

The Millikan Library at the California Institute of Technology is a nine-story, reinforced concrete building constructed in 1966 (11). The lateral load resistance in the N-S direction is provided by reinforced concrete shear walls and the resistance in the E-W direction is provided by a central elevator and stairwell core also of reinforced concrete. In addition, the structure possesses a reinforced concrete frame. The shear walls comprise the east and west faces of the building, whereas the north and south faces consist of precast concrete window-wall panels which are attached three per floor between reinforced concrete columns. It was determined from forced vibration tests of the structure during construction that these precast window-wall panels added appreciable stiffness to the structure for motions in the E-W direction. An exterior view of the building is shown in Figure 1 which also includes sketches of the foundation. More detailed information about the structure can be found in References 10 and 11.

Results of Vibration Tests Before the Earthquake

During the final stages of construction, the library was subjected to an extensive series of dynamic tests by J. Kuroiwa and one of the authors (10, 11). In these tests it was found that the fundamental period in the E-W direction was 0.66 secs. This value increased roughly 3% over the amplitude range of testing. The mode shape corresponding to this fundamental frequency was found from measurements taken at every other floor of the structure. In the vibration test the damping in the fundamental E-W mode varied between 0.7 and 1.5 percent of critical, increasing with the amplitude of response. Measurements of the foundation motions and motions on the nearby surface of the ground showed that the building responded nearly as if it were fixed at the foundation; rocking contributed less than one percent to the total roof motion of the structure, and foundation translation less than about two percent.

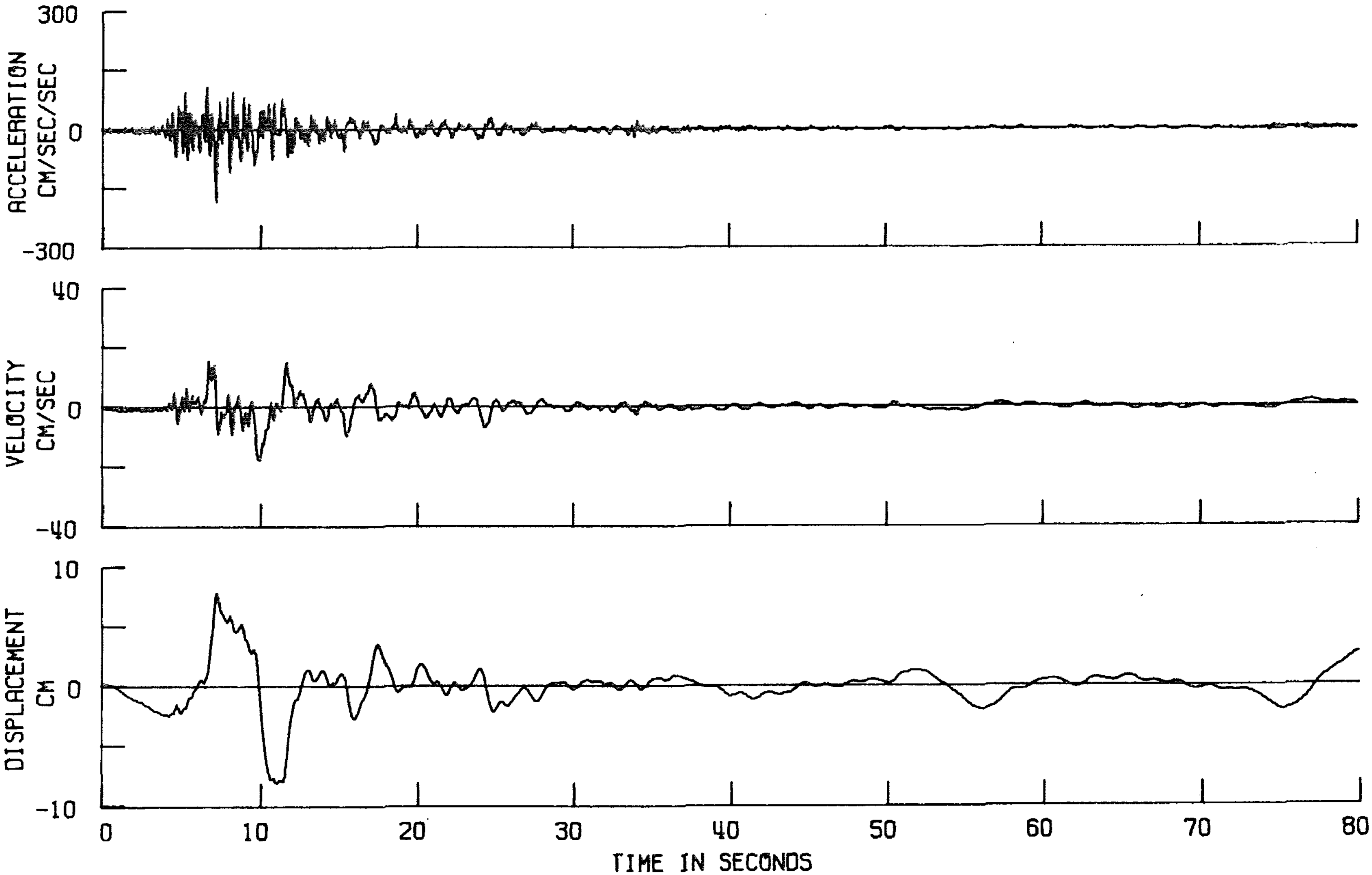
Within a few days after the earthquake an ambient vibration test was performed on the structure during which the fundamental E-W period was observed to be 0.80 secs (12, 14). Hence, there appeared to be a permanent change in the fundamental period of small vibrations in the E-W direction. It has been found that since the post-earthquake test, the structure has partially recovered and it exhibited a fundamental period of 0.73 secs in the E-W direction in December, 1972 (12).

Accelerograms Recorded During the Earthquake

Two accelerograms, one at the basement and one at the roof, were obtained at the Millikan Library during the San Fernando earthquake. The accelerograms and the calculated velocities and displacements are shown in Figures 2 and 3 (15, 16). In these figures, 80 secs of motion is shown, but not all of this motion is important to the present study. The first 40 secs of the accelerogram at the basement may be separated for discussion into two parts. The first part of the accelerogram (from 0 to about 15 secs) has a high acceleration level with a relatively high predominant frequency, whereas the second part of the record (from 15 to 40 secs) shows a relatively low acceleration level, and a lower predominant frequency. Comparing Figures 2 and 3, it seems that the different character of the response in Figure 3 in the early and latter parts of the record may be due to the different types of excitation that arrived during these two portions of time (17); there appears to be a larger fraction of surface waves in the latter portion of the basement accelerogram. The first part (0-15 secs) of the accelerogram shown in Figure 3 consists of a mixture of the first and second modes of response. The period of the second mode in the E-W direction is approximately 0.17 sec. During the second portion of the response (15-40 secs) the motion consists almost exclusively of the fundamental mode. Comparing the levels of measured acceleration at the base and at the roof, it appears that the ground motion was such as to excite the structure in a quasi-resonant fashion during the latter part of the record.

The displacement record in Figure 3 consists of a short-period (about 1.0 sec) portion and fluctuations at longer periods. Since the acceleration at the roof records the absolute motion of the structure, it is considered that the displacement record shows a combination of the motion of the structure with respect to the base, which is the motion of shorter period, superimposed upon a longer-period motion which represents the displacement of the foundation seen in Figure 2.

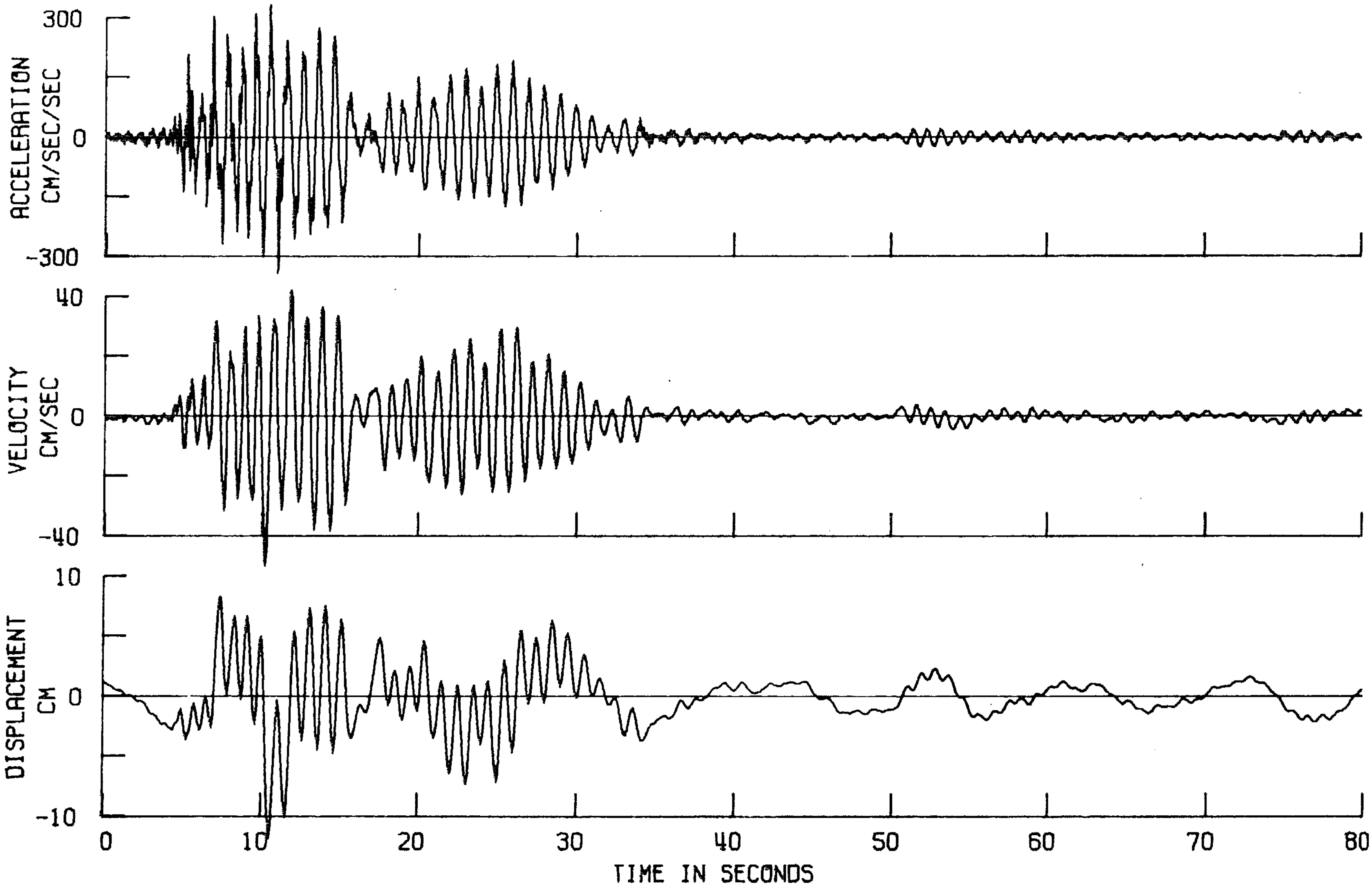
There are two major characteristics of the motion which are most apparent from examination of the records of earthquake response. First,



SAN FERNANDO EARTHQUAKE FEB.9,1971
MILLIKAN LIBRARY E-W DIRECTION BASEMENT RECORD

FIGURE 2

Recorded acceleration, and computed velocity and displacement for the basement motion, E-W direction.



SAN FERNANDO EARTHQUAKE FEB.9,1971
MILLIKAN LIBRARY E-W DIRECTION ROOF RECORD

FIGURE 3

Recorded acceleration, and computed velocity and displacement for the roof motion, E-W direction.

the fundamental period of the E-W vibration during the strong motion is about 50% longer than that measured at small amplitudes during vibration tests; it is clear from Figure 3 that the period of the E-W fundamental mode during the earthquake is near one second. Second, the records show that the library responded primarily in its fundamental mode in this direction. Although there is some vibration of the second mode apparent in the first part of the response, it is generally small with respect to the response of the fundamental mode. From these observations it was thought possible to consider the library to be a single-degree-of-freedom hysteretic structure responding to the earthquake, filtering or disregarding components of higher modes of response.

ANALYSIS OF RECORDED MOTIONS

Calculation of Relative Velocity and Displacement

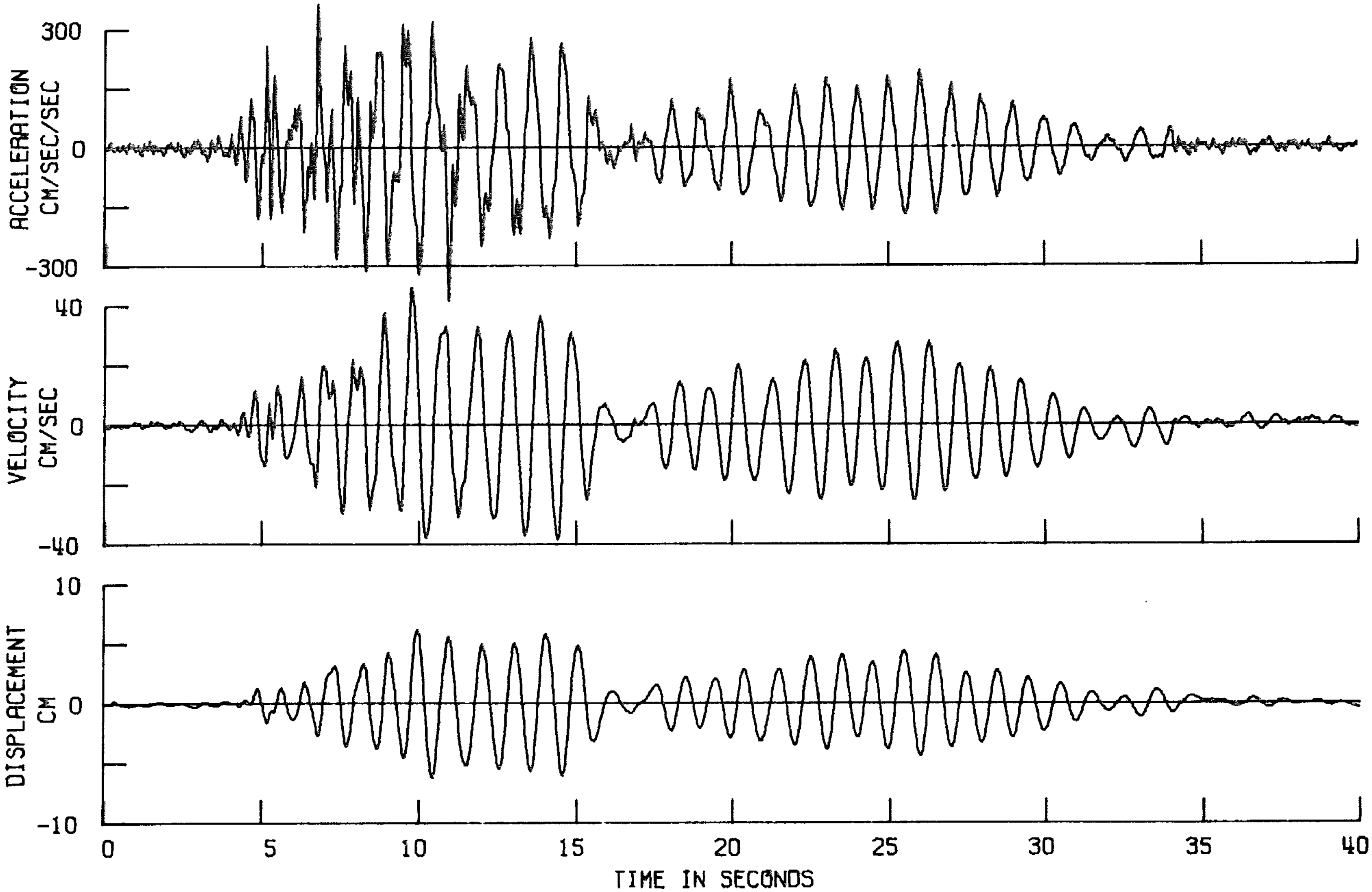
The calculation of relative velocity and displacement is required to determine the hysteretic character of the restoring force acting on the structure as a function of amplitude of response. Considering the library as a single-degree-of-freedom oscillator, the acceleration, velocity and displacement shown in Figure 3 may be considered as the absolute response of the oscillator, whereas those in Figure 2 may be considered as the base motion. Therefore, if the two recorded accelerograms have an accurate time correspondence, the relative velocity and displacement of the oscillator can be obtained by subtracting the calculated ground velocity and displacement from the calculated values of velocity and displacement obtained from the record measured on the roof.

Fortunately, the two accelerograms were recording a common time signal and were, in fact, a part of a more extensive network (18) which included accelerographs at the Jet Propulsion Laboratory, Millikan Library and the Caltech Seismological Laboratory.

When the calculated values of velocity and displacement were subtracted from each other to obtain the relative motion, it was found in preliminary analyses that the relative displacement included long fluctuations with a period of about 11 secs. It was subsequently pointed out by T. C. Hanks that these were due to a processing error in the digitizing of some accelerograms, which has since been corrected. To eliminate this 11-second period motion from the records analyzed in this study, a low-pass filter proposed by Jennings, Housner and Tsai (19) was employed. The subtracted and corrected relative acceleration, velocity and displacement are plotted in Figure 4. Comparing Figures 3 and 4 (which are at different time scales) it is seen that the changes in the acceleration are slight, but the smoothing process of the integration, the subtraction of the long-period ground displacement, and the elimination of the digitizing error, have led to comparatively smooth curves for relative velocity and displacement. Some possible contributions of the second mode to the acceleration and relative velocity can be seen, but the relative displacement is essentially only that of the fundamental mode. It should be noted in Figure 4 that the motions beyond 40 secs have no longer been included in the analysis.

Analysis of Natural Frequencies and Amplitudes

From the relative displacement shown in Figure 4 and replotted in Figure 5, it is easy to see that the period of the motion is much longer than the period of vibration exhibited at small amplitudes. To investigate the nature of the nonlinearity of the restoring force, the period and corresponding amplitude of each whole cycle of displacement were measured from Figure 5 and plotted in Figure 6. The number of each point in Figure 6 corresponds



SAN FERNANDO EARTHQUAKE FEB.9,1971
MILLIKAN LIBRARY E-W DIRECTION

FIGURE 4

Relative values of acceleration, velocity and displacement of the roof with respect to the basement.

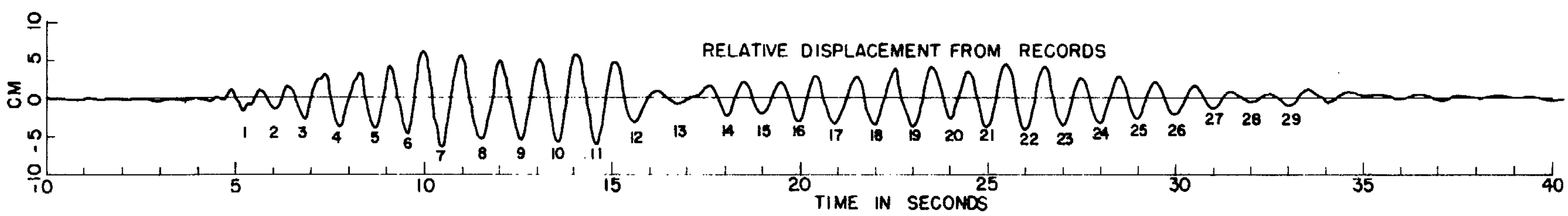


FIGURE 5

Relative displacement of the roof with respect to the basement.
Numbers index cycles of the fundamental mode.

to the number of the cycle as indicated in Figure 5. The results obtained from the vibration tests before and after the earthquake are also plotted in Figure 6. The points in Figure 6 show considerable scatter, which is expected in a measure this crude, and it is hard to find clear relations in the figure. However, the points numbered 1 to 11 and the results of the tests before the earthquake suggest an approximately linear relation between the amplitude and period of vibration, with the larger amplitudes corresponding to the longer-period motion. The remaining points, numbers 12 to 29, are scattered about one second over a fairly wide range, but above the approximately linear band shown by points 1 to 11.

These results suggest that the library may have behaved like one hysteretic structure up until about 15 secs, and then changed to a different hysteretic structure. This is also consistent with the observed loss of structural stiffness indicated by the post-earthquake vibration test. To investigate this suggestion further, the measured periods of vibration are plotted on the time axis in Figure 7, which also includes, as lines, the results from the vibration tests before and after the earthquake. It is seen from this figure, which also shows considerable scatter, that the natural period tends to increase gradually until about 14 secs. Points number 12 and 13 show unusually long periods but these points may be subject to more error than others as the amplitude of response is quite small (Figure 5). After these points, most of the values fluctuate around one second. Similar trends were obtained by F. E. Udwadia and M. D. Trifunac from their analysis of the accelerograms using Fourier transform techniques (12, 13). The results from their work confirm that the fundamental period of vibration in the E-W direction increased about 50% during the first part of the strong shaking, and remained at about

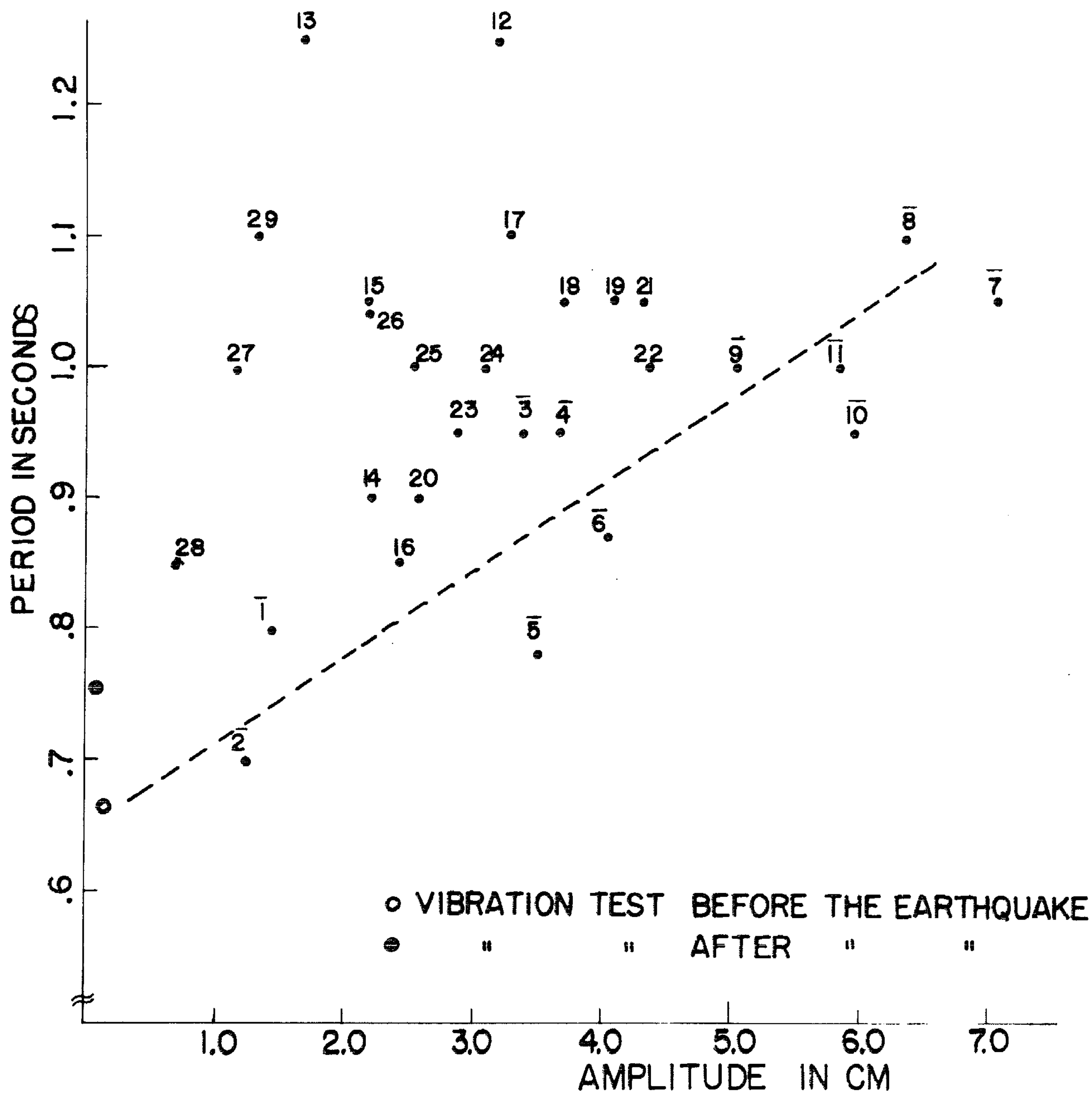


FIGURE 6

Plot of period of displacement cycles vs. amplitude of motion. Numbers are indices shown in Figure 5.

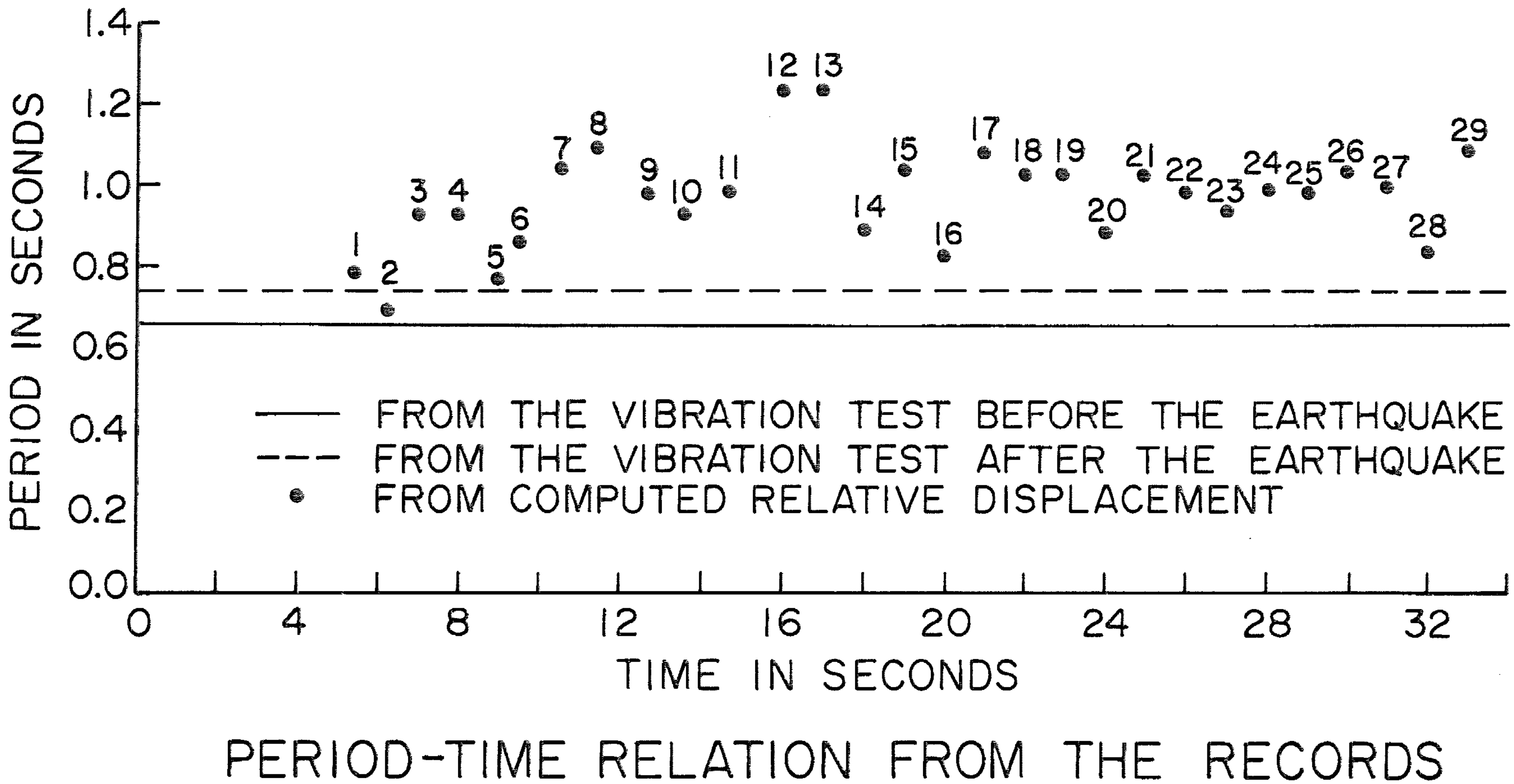


FIGURE 7

Plot of period of displacement cycles vs. time. Numbers are indices shown in Figure 5.

one second for the rest of the first 40 secs of response, even though the amplitude of the response decreased. Their analysis also showed unusual behavior at about $t = 15$ secs and, in the period of weak response from 40 to 80 secs, a tendency for the period to shorten from 1.0 sec to values in the range of 0.8-0.9 secs.

Experimental Hysteretic Diagrams

The analysis of the previous section identifies the nonlinearity of the restoring force as the reason for the lengthening fundamental period observed during the earthquake. In this section, the time-dependence of the hysteretic behavior of the library is studied by plotting the measured values of acceleration against the calculated values of relative displacement.

Consider the equation of motion of a single-degree-of-freedom system excited by an earthquake:

$$F(x, \dot{x}) = -M[\ddot{x} + \ddot{z}] \tag{1}$$

in which $F(x, \dot{x})$ represents the nonlinear restoring force due to relative velocity $\dot{x}$ and displacement x; M is the mass and $\ddot{z}$ is the ground acceleration. Equation 1 shows that the total restoring force divided by the mass is the negative of the absolute acceleration. Using this relation, a preliminary version of the hysteretic response of the library was obtained by plotting the relative displacement shown in Figure 4 vs. the absolute acceleration shown in Figure 3. This trajectory, plotted every 0.02 secs, gave a reasonable estimate of the first-mode hysteresis of the library during the second portion of the response, because the first mode of the vibration predominates at this time. However, the first portion of the response (0 to 15 secs) showed marked fluctuations along the trajectory of the supposed first-mode hysteresis. This fluctuation was thought to be the result of the non-negligible contributions of the second mode of vibration of the library, which is discernible in this part of the acceleration records.

Because of the assumptions of the study it was considered appropriate to eliminate the effect of the second mode of vibration as well as any higher modes that may have participated in the response. If this could be done, the desired trajectory between the absolute acceleration and the relative displacement of the fundamental mode would be obtained. The simple low-pass filter (19) mentioned above was again used to eliminate these higher mode responses from the absolute acceleration record and then the relative velocity and displacement were calculated. These curves are shown in Figure 8, which can be compared with the absolute acceleration in Figure 3 and the relative velocity and displacement in Figure 4. It can be seen from this comparison that the response of the higher modes has been greatly diminished, but not completely eliminated, especially in the region from about 5 to 8 secs. Using the results shown in Figure 8, the trajectory between the first mode absolute acceleration, which is porportional to the restoring force by Equation 1, and the relative displacement was plotted every 0.02 secs and is given in Figure 9. In plotting these trajectories it was found that there was a small phase error of about .04 to .06 secs between the absolute acceleration and the relative displacement. This phase error significantly affected the shape of the trajectories and, unless corrected, some of the trajectories indicated negative hysteretic damping. By close examination of the digitized data, the amount of this phase error was found to differ over the first 12 seconds of the response when compared to the part after 12 secs. The source of this small phase error could not be identified, but it is small enough that it is a possibility that it is a phase difference in the

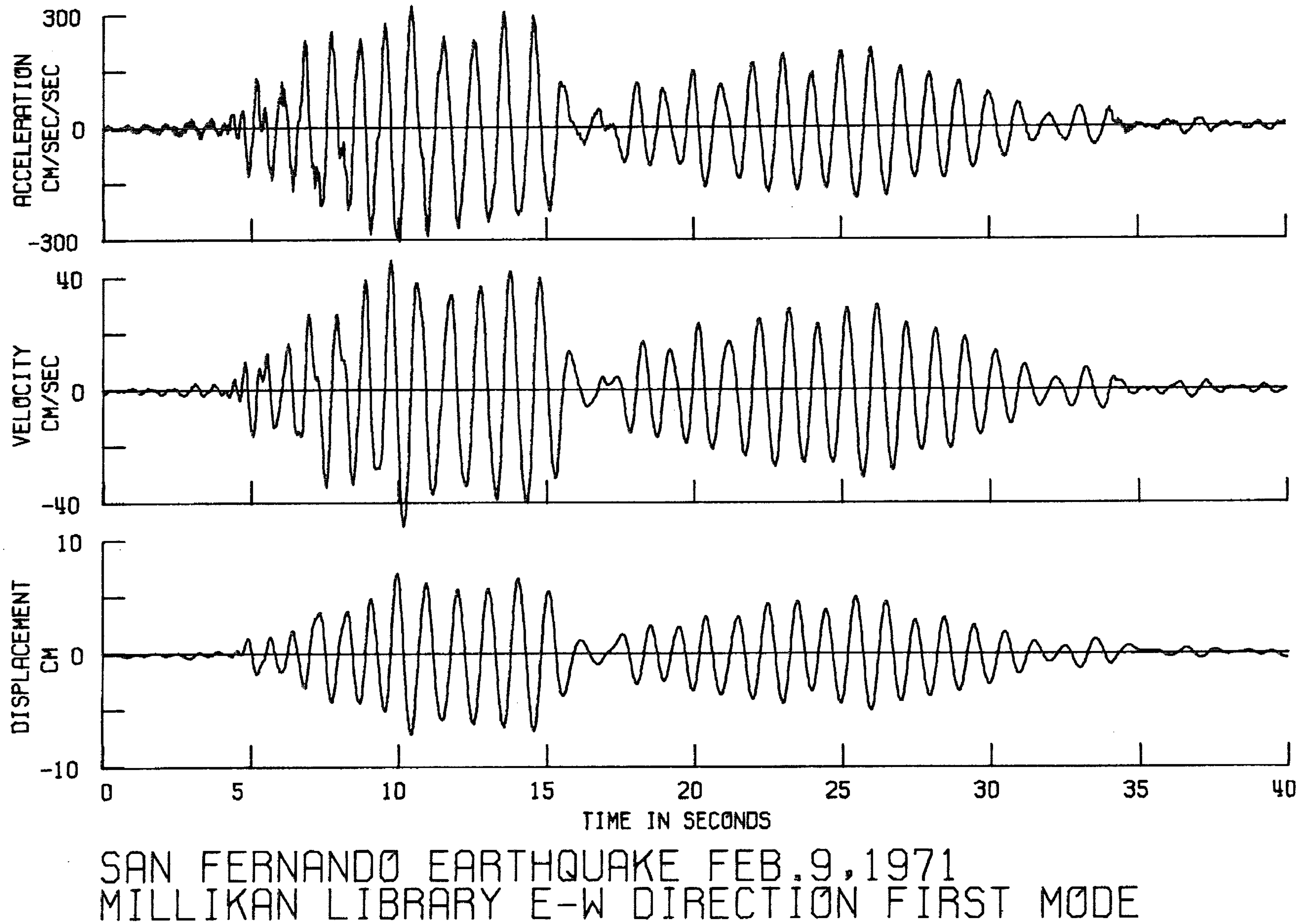
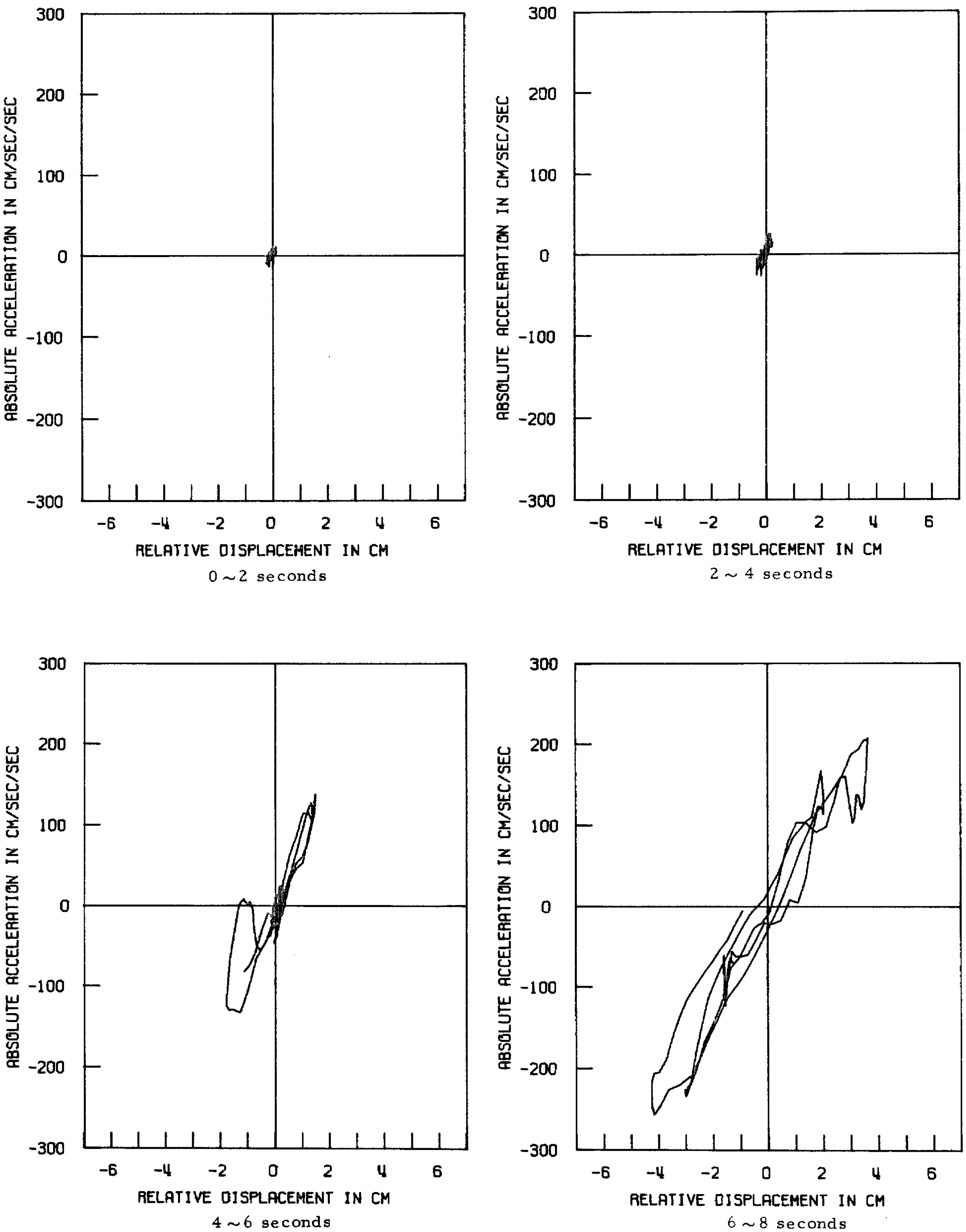


FIGURE 8

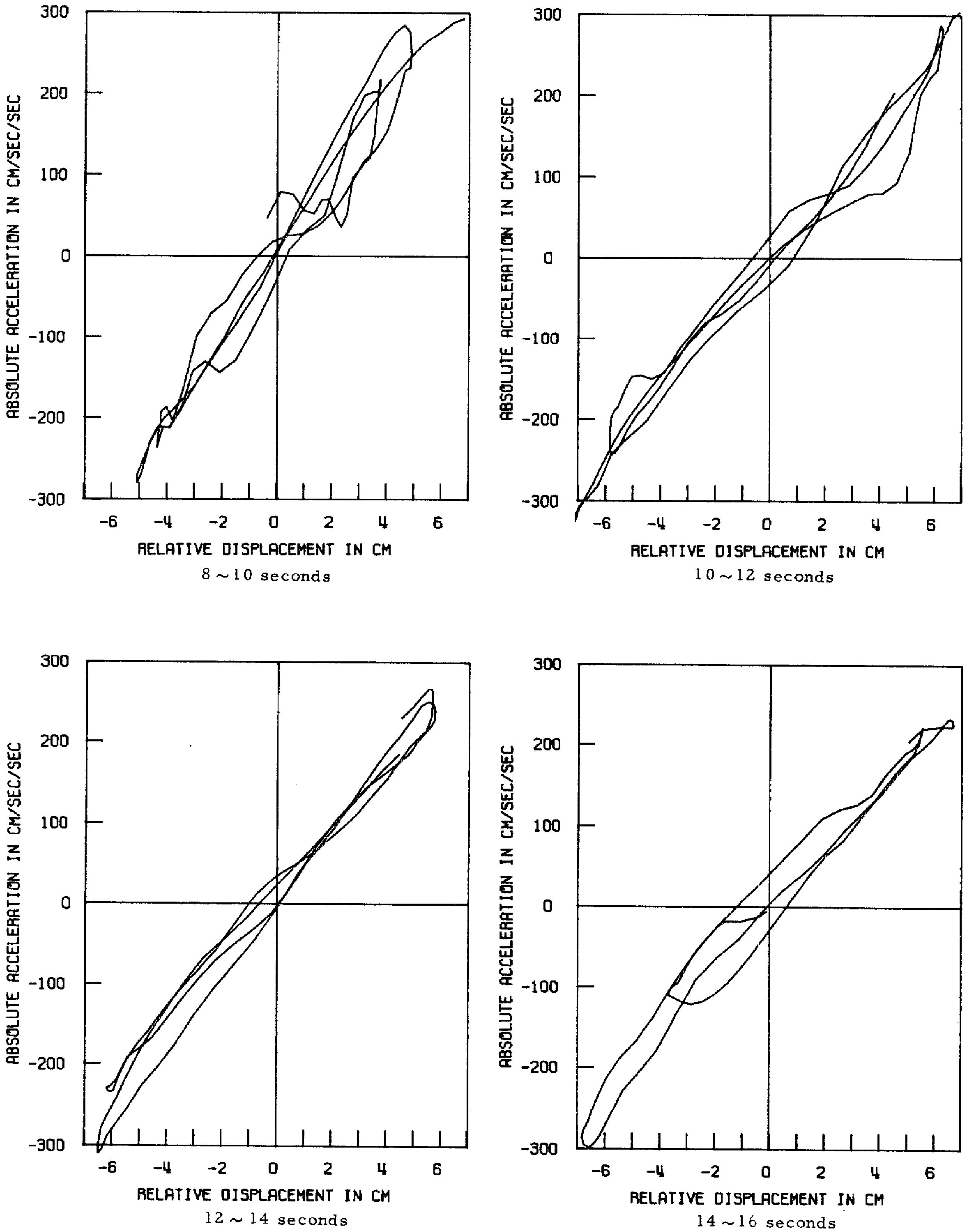
Absolute acceleration, relative velocity and relative displacement, filtered to remove response of higher modes



FIRST MODE HYSTERESIS OF MILLIKAN LIBRARY FROM RECORD

FIGURE 9a

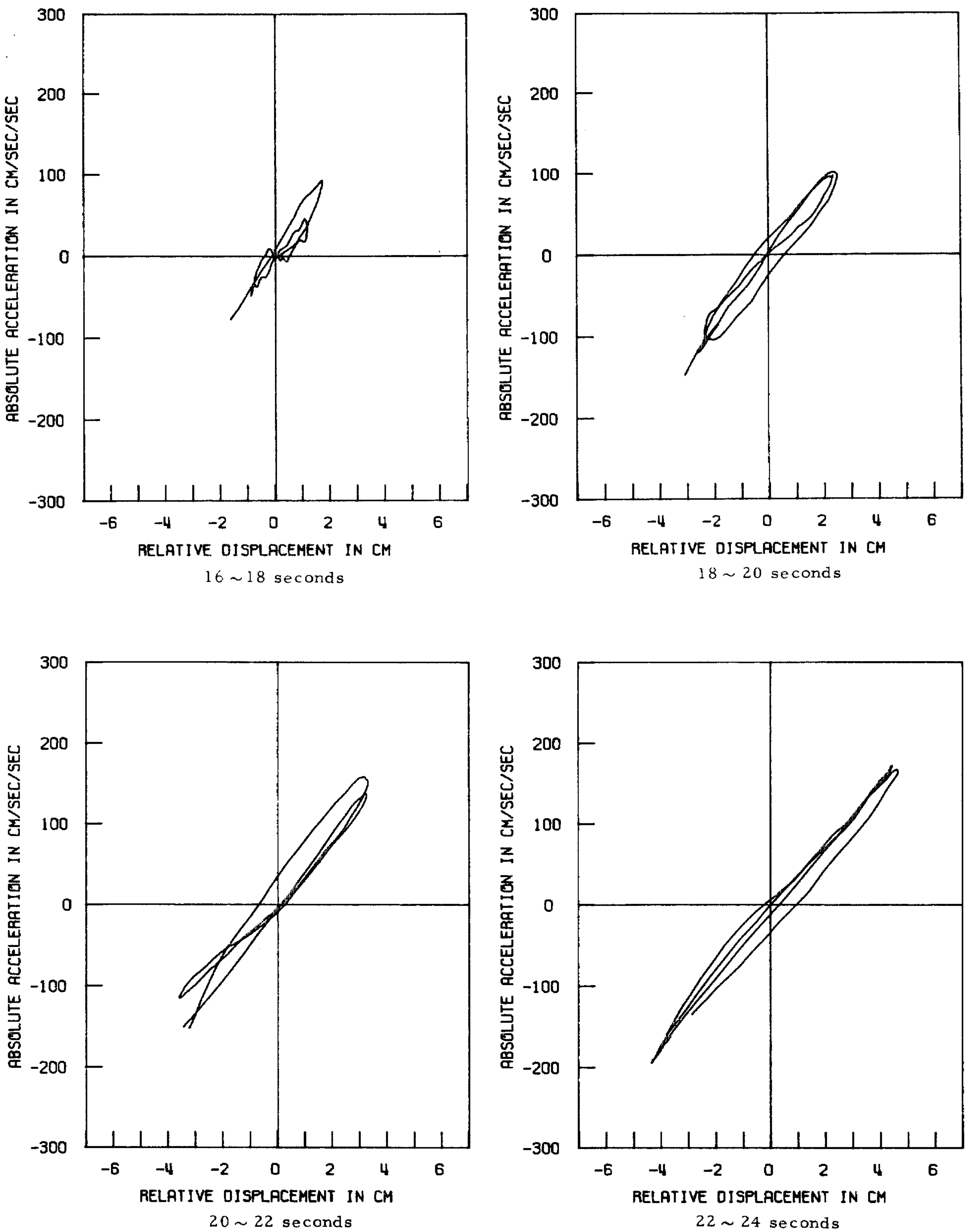
Hysteretic response of single-degree-of-freedom oscillator modelling fundamental mode, as determined from recorded motions.



FIRST MODE HYSTERESIS OF MILLIKAN LIBRARY FROM RECORD

FIGURE 9b

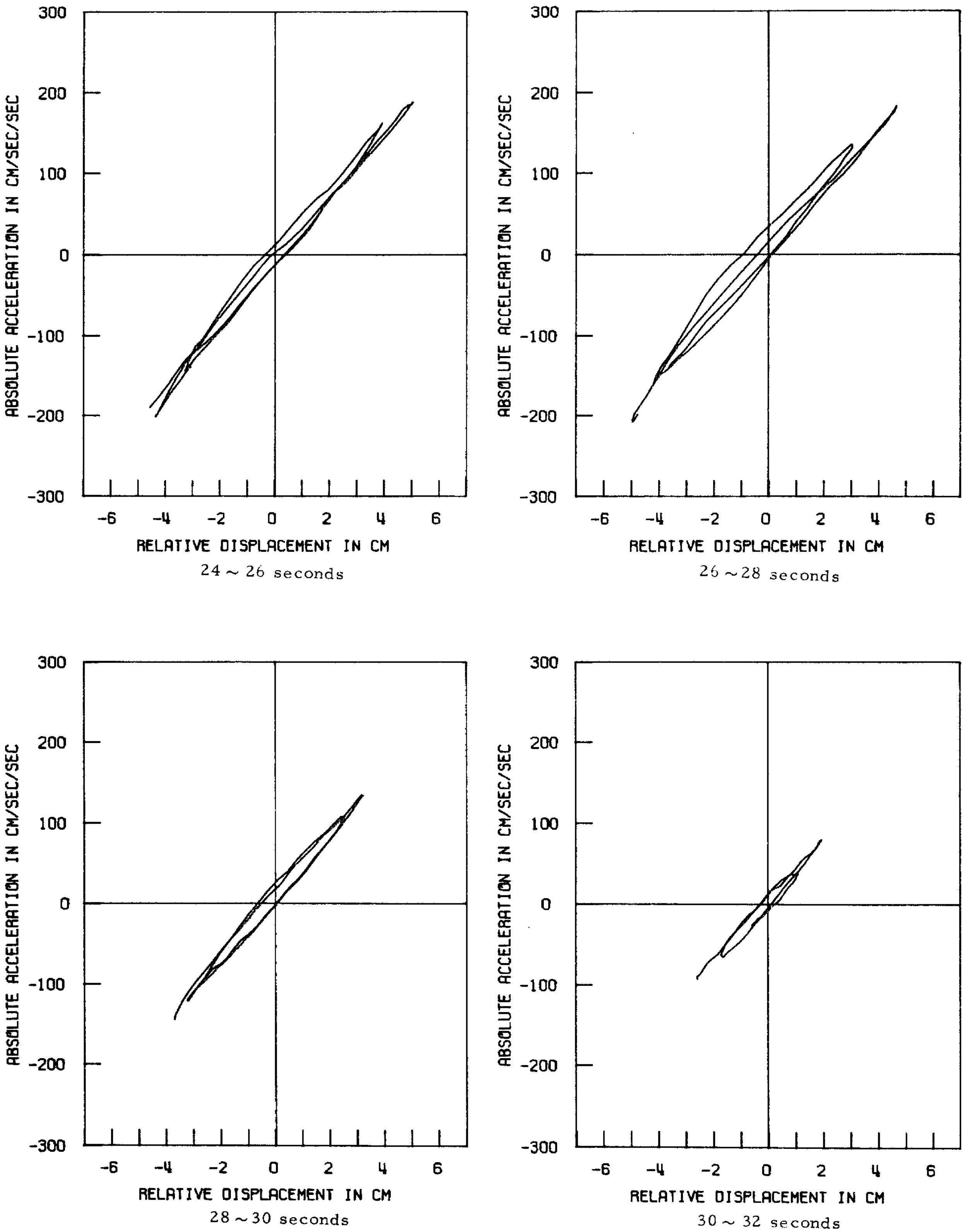
Hysteretic response of single-degree-of-freedom oscillator modelling fundamental mode, as determined from recorded motions. (Cont'd.)



FIRST MODE HYSTERESIS OF MILLIKAN LIBRARY FROM RECORD

FIGURE 9c

Hysteretic response of single-degree-of-freedom oscillator modelling fundamental mode, as determined from recorded motions. (Cont'd.)



FIRST MODE HYSTERESIS OF MILLIKAN LIBRARY FROM RECORD

FIGURE 9d

Hysteretic response of single-degree-of-freedom oscillator modelling fundamental mode, as determined from recorded motions. (Cont'd.)

in the digitization, which cannot be expected to be much more accurate than about .04 to .06 secs. Other possibilities include Instrument malfunction around $t = 12$ secs or a small error in phase that might have been introduced because of the application of the filters to the record.

To adjust for the phase error, the time history of the relative displacement was shifted to match the peak value of the absolute acceleration during the two parts of the response. This was done because the maximum restoring force should occur at the same time as the maximum relative displacement for the small values of viscous damping associated with the library. At the beginning of the response, from 0 to 4 secs as shown in Figure 9a, the hysteretic properties of the library are not clear because of the small amplitudes. As is well known, the tangent of the trajectory is equal to the square of the fundamental natural frequency for a structure that responds essentially in the linear range. The slope of the trajectory from 0 to 4 secs appears to be close to that of a linear structure with a natural period of 0.66 secs (for which the tangent value is $90/sec^2$), indicating that the library was vibrating at the beginning of the earthquake with the fundamental period found during the pre-earthquake vibration tests.

The slope of the trajectory is still steep from 4 to 6 secs as shown in Figure 9a. However, the plots show more hysteresis due to the high response levels. There is a large loop on the minus side of the trajectory and afterwards there is a sharp drop in the restoring force, perhaps indicating a sudden change in some structural elements due to the strong vibration. From 6 to 8 secs the slopes of the hysteresis loops have become less and the areas of the hysteresis loops have become larger. There are also some short-period fluctuations along the supposed first-mode hysteresis loops which are thought to be the results of the incomplete filtering of the absolute accelerogram as discussed above. It is also possible that these fluctuations represent small errors in the calculation, which appears to be a sensitive one. Figure 9b shows the response from 8 to 10 secs, and it is seen that there are still some fluctuations and sudden changes in the restoring force but, in general, the loops are becoming smoother. The slopes of the hysteresis loops are clearly less than during the early part of the earthquake, and the area of the hysteresis loops is still large. As seen in the same figure, the slope of the hysteresis loops from 10 to 12 secs are almost as soft as the stiffness of the linear structure with a natural period of one sec (a tangent value of $39/sec^2$). There is a suggestion, however, that the areas of the hysteresis loops during the period from 10 to 12 secs are less than those for 6 to 8, or 8 to 10 secs. There are still fluctuations in the trajectory which may be associated with the second mode of response. From 12 to 14 secs the areas of the hysteresis loops have clearly become smaller, suggesting that the energy dissipation capacity of the library at this amplitude has decreased because of the previous vibrations. The hysteresis loops are also noticeably smoother, presumably due to the predominance of the fundamental mode of vibration. The remaining portion of the response, from 14 to 32 secs (figures 9b, c, and d) shows that the library continues to exhibit a softer restoring force with a relatively smaller energy dissipation capacity, when compared to the earlier response. This is true even though the response level is decreasing. Comparing the responses between 4 to 6 secs and between 28 to 30 secs, which have about the same absolute acceleration level, it is seen that there has been a degradation of the stiffness of the structure. It is also seen, from comparing the two figures for the periods from 6 to 8 secs, and from 24 to 26 secs, that the energy absorbing capacity of the library has changed during the earthquake response.

The overall indication gained from Figure 9 is that the library lost not only some of its stiffness, but also some energy dissipation capacity due to the large amplitude response during the first part of the earthquake. This nonstationary characteristic of the hysteretic behavior of the library agrees, in principle, with those of simple theoretical models of deteriorating structures, although the details of the hysteretic behavior are somewhat different from the theoretical models so far suggested.

It was thought desirable to estimate more precisely the loss of stiffness and energy absorbing capacity evidenced during the response. The stiffness of the library during each full cycle of relatively large amplitude has already been estimated and is shown in Figure 7. To make a similar study of the nonstationary behavior of the energy absorbing capacity, the hysteresis loops were used to estimate an equivalent viscous damping factor for each full cycle of response.

In this study the equivalent viscous damping factor h_{eq} was defined by equating the energy dissipated by hysteresis to that dissipated by viscous damping.

$$\oint F(\mu, \dot{\mu}) d\mu = \oint 2h_{eq} \, \omega_{eq} \dot{\mu} \, d\mu \tag{2}$$

in which $F(\mu, \dot{\mu})$ is the restoring force; μ, $\dot{\mu}$ are the relative displacement and velocity respectively, and ω_{eq} is the equivalent natural frequency measured from that portion of the response. To evaluate the right-hand side of Eq. 2, it was assumed that over a cycle the amplitude of the response was a slowly varying sine wave, i. e.

$$\dot{\mu}(t) = -\omega_{eq} \, \mu_o(t) \sin [\omega_{eq} t + \phi(t)] \tag{3}$$

in which $\phi(t)$ is a slowly varying phase angle.

From Eqs. 2 and 3 the equivalent viscous damping factor h_{eq} is obtained as

$$h_{eq} = \frac{1}{2\pi \omega_{eq}^2 \mu_o^2} \oint F(\mu, \dot{\mu}) d\mu \qquad (4)$$

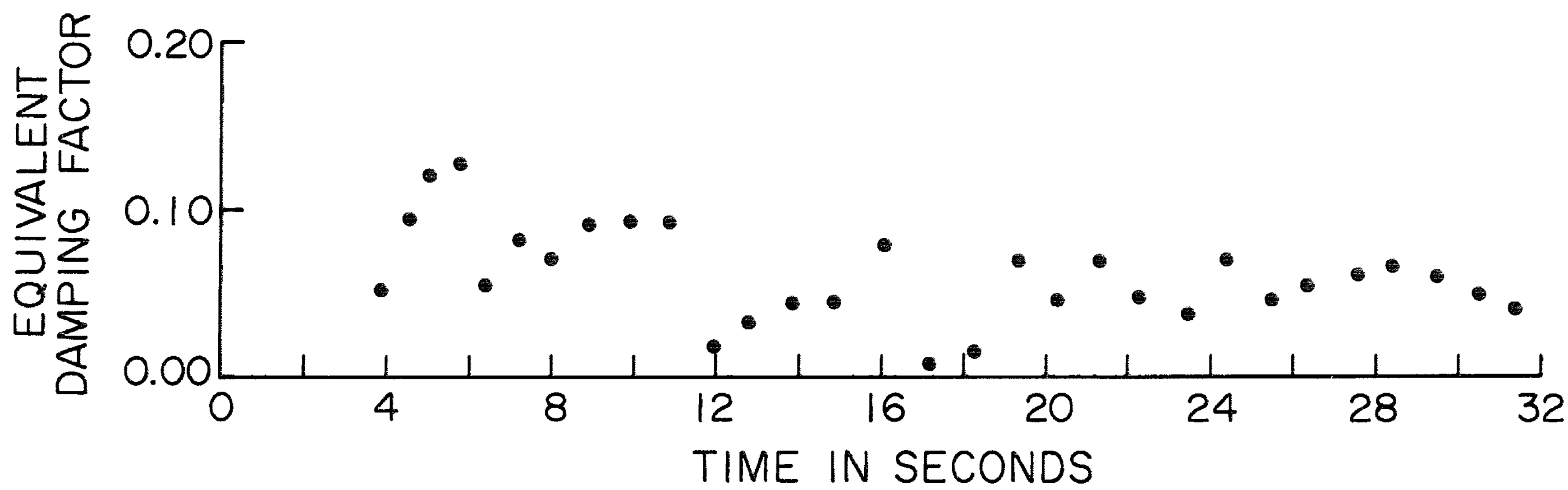
in which μ_o is the measured amplitude of the relative displacement and $\oint F(\mu, \dot{\mu})$ is evaluated from the hysteresis loops given in Figure 9.

The equivalent viscous damping factors calculated this way are plotted for each full cycle in Figure 10. From the nature of the assumptions involved and the inherent errors, it is not expected that this would be a precise calculation. Figure 10 indicates that the library showed about 8 to 10% of critical damping from about 4 to 10 secs at which time the amplitude of the response reaches a maximum value. After 10 secs the energy absorbing capacity shows a reduction, which is consistent with the suggestion that a relatively sudden change in the energy absorbing capacity took place at about the time of maximum response. As pointed out above, the only observable earthquake effects on the structure were small cracking in the plaster in the vicinity of the mounts of the precast window wall panels. It is one possibility that the working loose of these mountings was the cause of the observed behavior.

ANALYTICAL MODELS OF FIRST-MODE RESPONSE

A Stationary, Equivalent Linear Model

Before attempting to model the response by nonlinear hysteretic behavior, a simple linear model was tried, both to establish a base for further comparisons and to investigate the capabilities of this simplest possible approach. In order to model the first mode of the library as a



EQUIVALENT DAMPING FACTOR FROM THE FIRST MODE HYSTERESIS

FIGURE 10

Equivalent damping factors determined from analysis of measured response.

simple oscillator it was necessary to calculate the participation factor of the fundamental mode. This was done using the experimentally determined values obtained in the vibration tests. As indicated in the Appendix, the input acceleration level to the equivalent linear oscillator was adjusted by the participation factor of the fundamental mode and by the weighting factor for the response of the roof.

The period of the equivalent linear model was taken as 1.0 sec in agreement with the second part of the response shown in Figure 7. The equivalent damping factor was chosen to be 5% of critical damping, a representative value taken from Figure 10. It might be noted that approximately 2% of this 5% can be associated with viscous-like damping measured in the pre-earthquake vibration tests.

Using this simple linear model, the absolute acceleration, relative velocity and relative displacement were calculated and plotted in Figure 11. During the early part of the response from 0 to 15 secs the calculated response in Figure 11 does not coincide with the first mode response shown in Figure 8. The difference is particularly noticeable around 10 secs, where the calculated response is decreasing, whereas the measured response is growing and showing its maximum value. The reason for this discrepancy is that in the beginning of the vibration the library has a fundamental period of about 0.66 secs, whereas the simple linear model has a period of one second throughout the response. In addition, the assumed dissipation value of 5% is less than actually shown by the library during that early portion of the response. The coincidence of the calculated response in Figure 11 and the first mode response in Figure 8 is much better

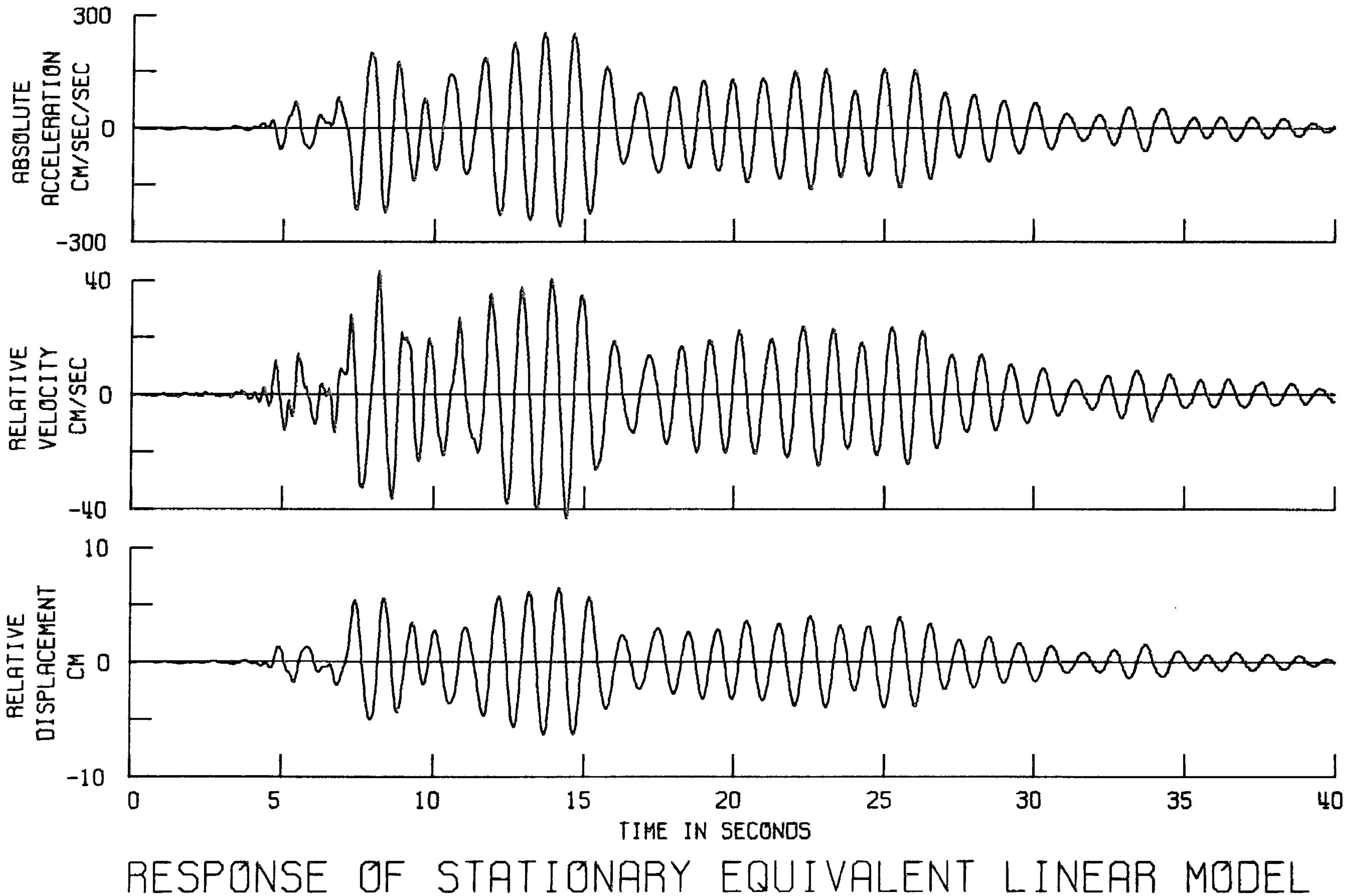


FIGURE 11

Absolute acceleration, relative velocity and relative displacement of the stationary linear model.

during the second part of the response, from 15 to 40 secs. In particular, the phase difference is very small. The simple linear model works well for this portion of the response, which is consistent with the smooth hysteresis loops shown in Figure 9, and the generally constant value of energy dissipation as indicated by Figure 10.

From these results it can be said that this simple linear model of the structure gives good agreement only for the portion of the response between 15 and 40 secs, the portion of the response over which structural parameters do not change significantly. The simple linear model does give a reasonably good estimate of the maximum response of the structure, and may therefore be useful from the point of view of design. From the point of view of research, however, it would seem that much better agreement could be obtained using a more detailed model of the hysteretic behavior.

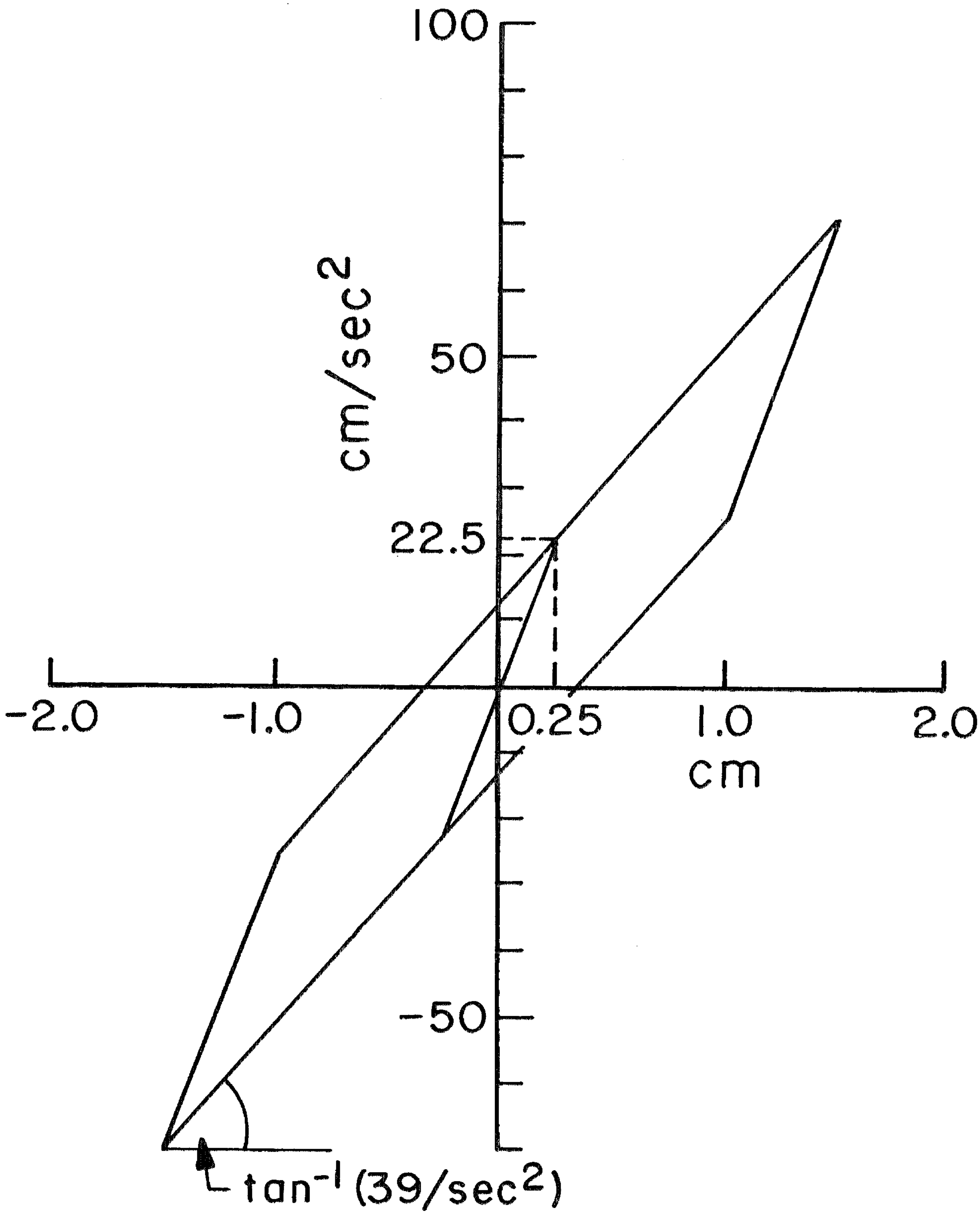
Stationary, Bilinear Model

A stationary, bilinear hysteretic model was adopted in this section to represent the nonlinear hysteretic characteristics of the restoring force of the structure. Considering the shape of the hysteresis loops given in Figure 9, and the trends in the equivalent linear parameters shown in Figures 7 and 10, the bilinear model selected was chosen to have a small yield displacement with respect to the maximum response, and a relatively steep second slope. The yield level of this model was chosen to fit the observed behavior and does not indicate yielding in the structural frame of the library. A hysteretic model with these parameters will, for large deflections, show a small amount of energy dissipation and an equivalent natural frequency which is almost the same as that indicated by the second slope of the hysteretic diagram. The first slope of the bilinear hysteretic model was chosen to give a natural period of 0.66 secs, whereas the second slope was chosen to correspond to a linear restoring force for a structure with a natural period of 1.0 secs. The transition point between these two slopes was set at 0.25 cm, which is only slightly larger than the maximum displacement during the vibration experiments. A typical hysteresis loop for a structure of this type is shown in Figure 12.

The calculated response of this bilinear hysteretic structure subjected to the recorded base acceleration is shown in Figure 13, which gives the absolute acceleration, relative velocity and relative displacement of the oscillator. The calculated hysteretic behavior comparable to Figure 9 is plotted in Figure 14. The response values plotted in Figure 13 show very poor agreement with those from the first-mode response, Figure 8, except for the phase in the period from 12 to 17 secs. Comparing the hysteretic response from 4 to 6 secs, as shown in Figures 9a and 14a, it is seen that the bilinear model gives a stiffness of the restoring force that is too low. Also it is seen from Figures 9b, c and 14b, c that the bilinear relation shows too much hysteretic damping from 8 to 24 secs. This is consistent with the calculated response being smaller than the measured response during this interval. These comparisons indicate that a satisfactory description of the response by a stationary hysteretic model is unlikely and that better agreement could be attained using a nonstationary mode.

<u>Nonstationary, Equivalent Linear Model</u>

It was seen previously that stationary models of the fundamental mode gave only limited agreement with response measured during the earthquake. In this section, a nonstationary, equivalent linear model, which changes its structural parameters at selected points during the response, was tried to see if the agreement could be improved. This was done both to check the



STATIONARY BILINEAR MODEL

FIGURE 12

Typical hysteresis loop for the stationary, bilinear hysteretic model.

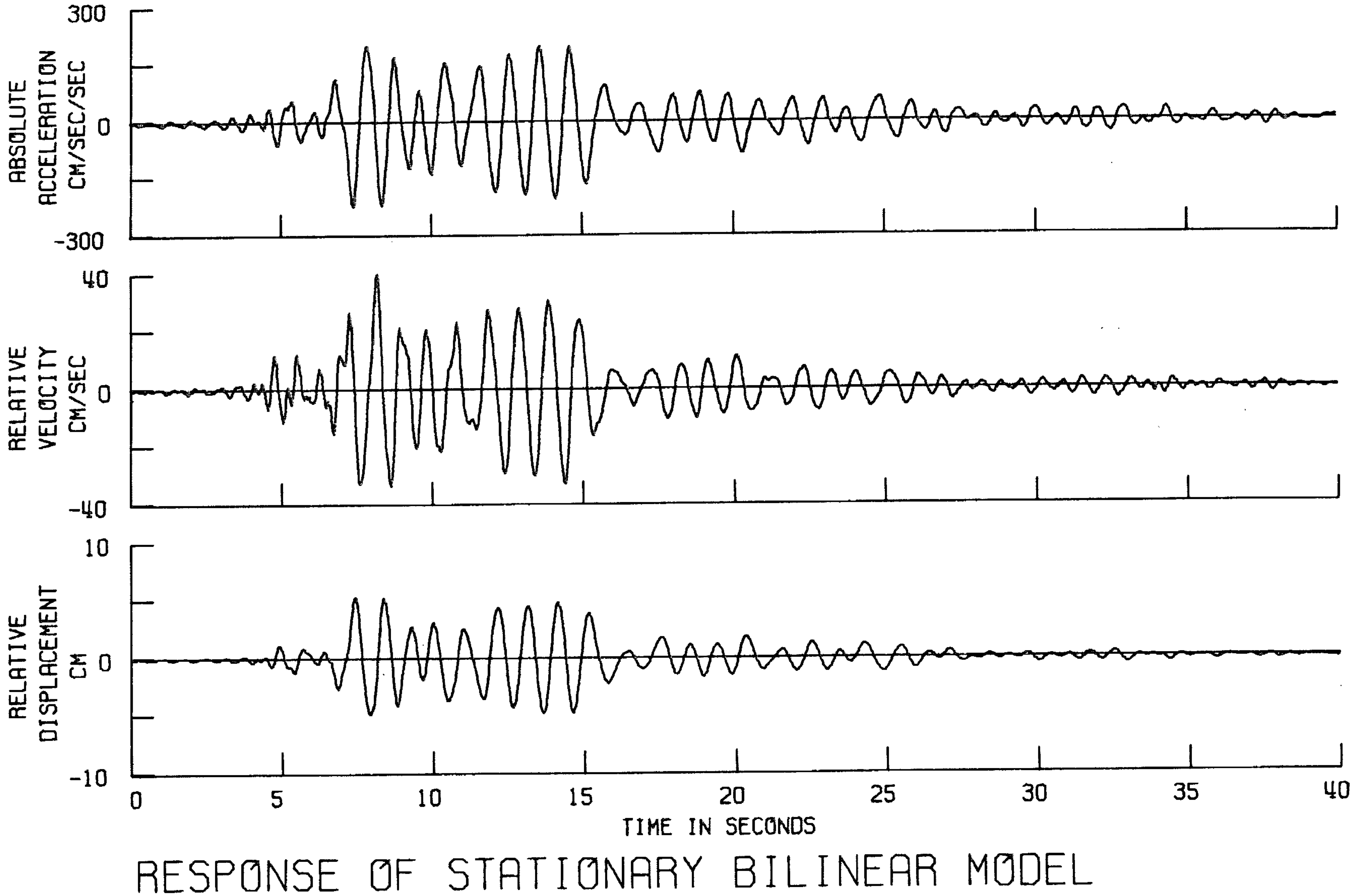
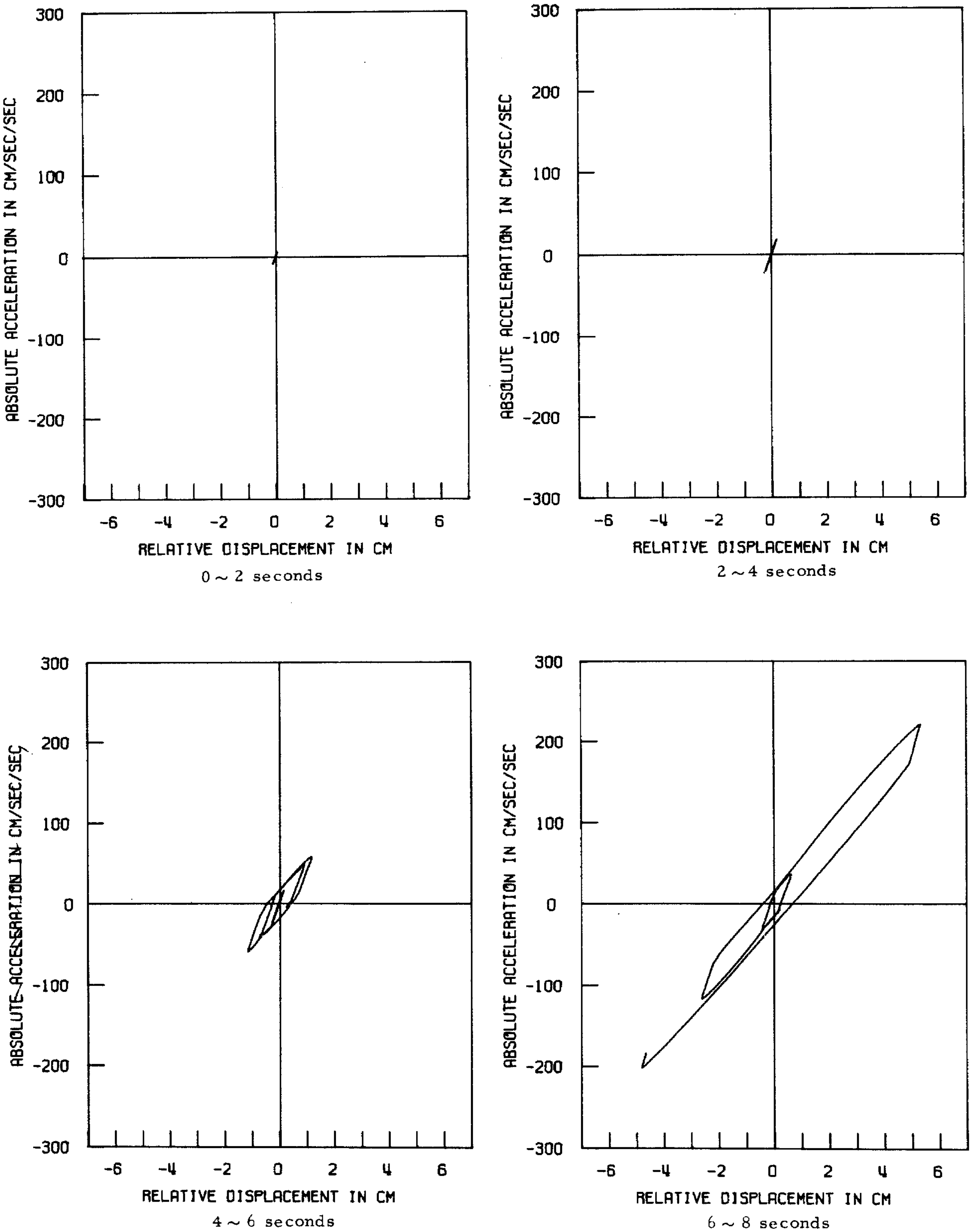


FIGURE 13

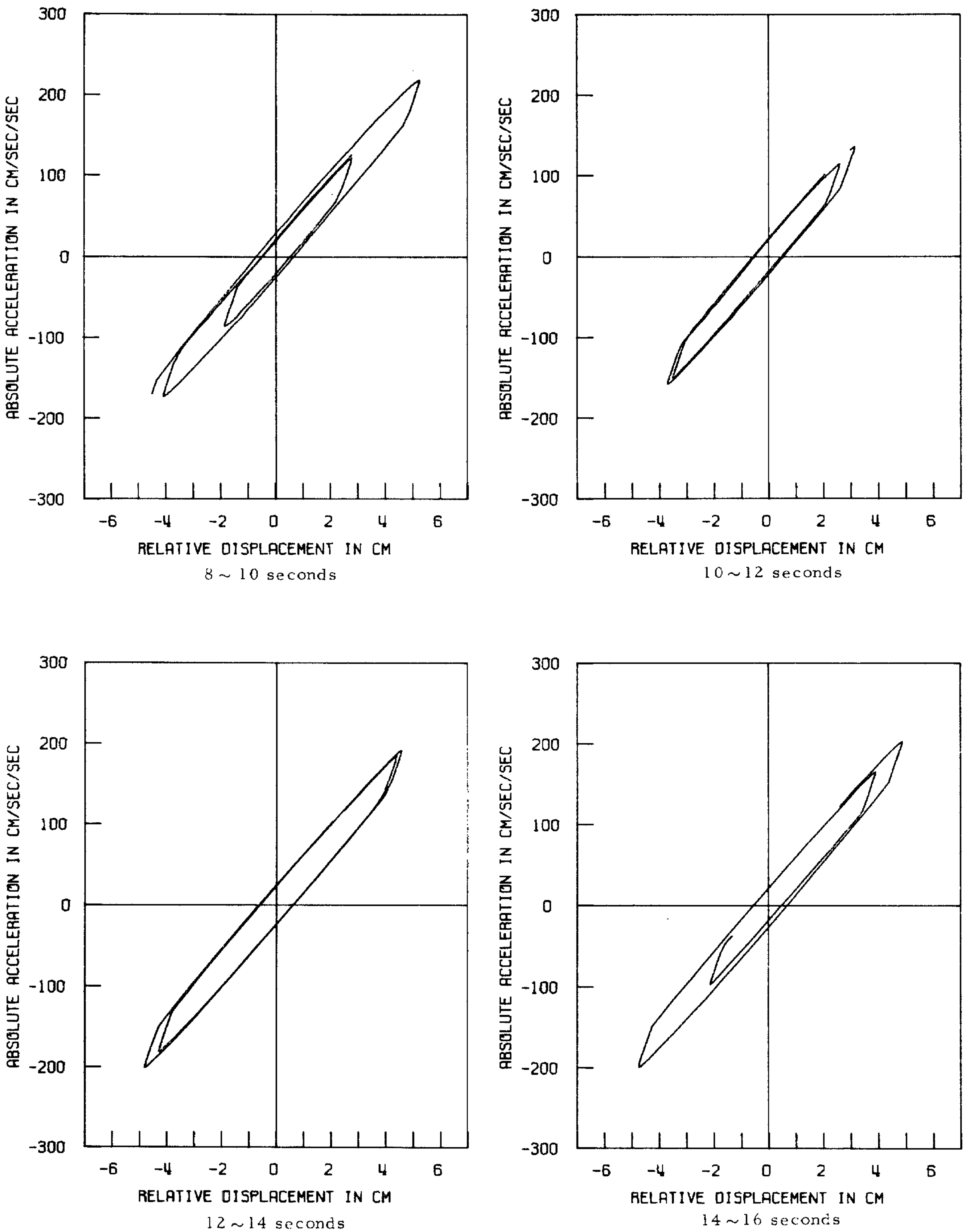
Absolute acceleration, relative velocity and relative displacement of the stationary, bilinear hysteretic model.



HYSTERETIC RESPONSE OF STATIONARY BILINEAR MODEL

FIGURE 14a

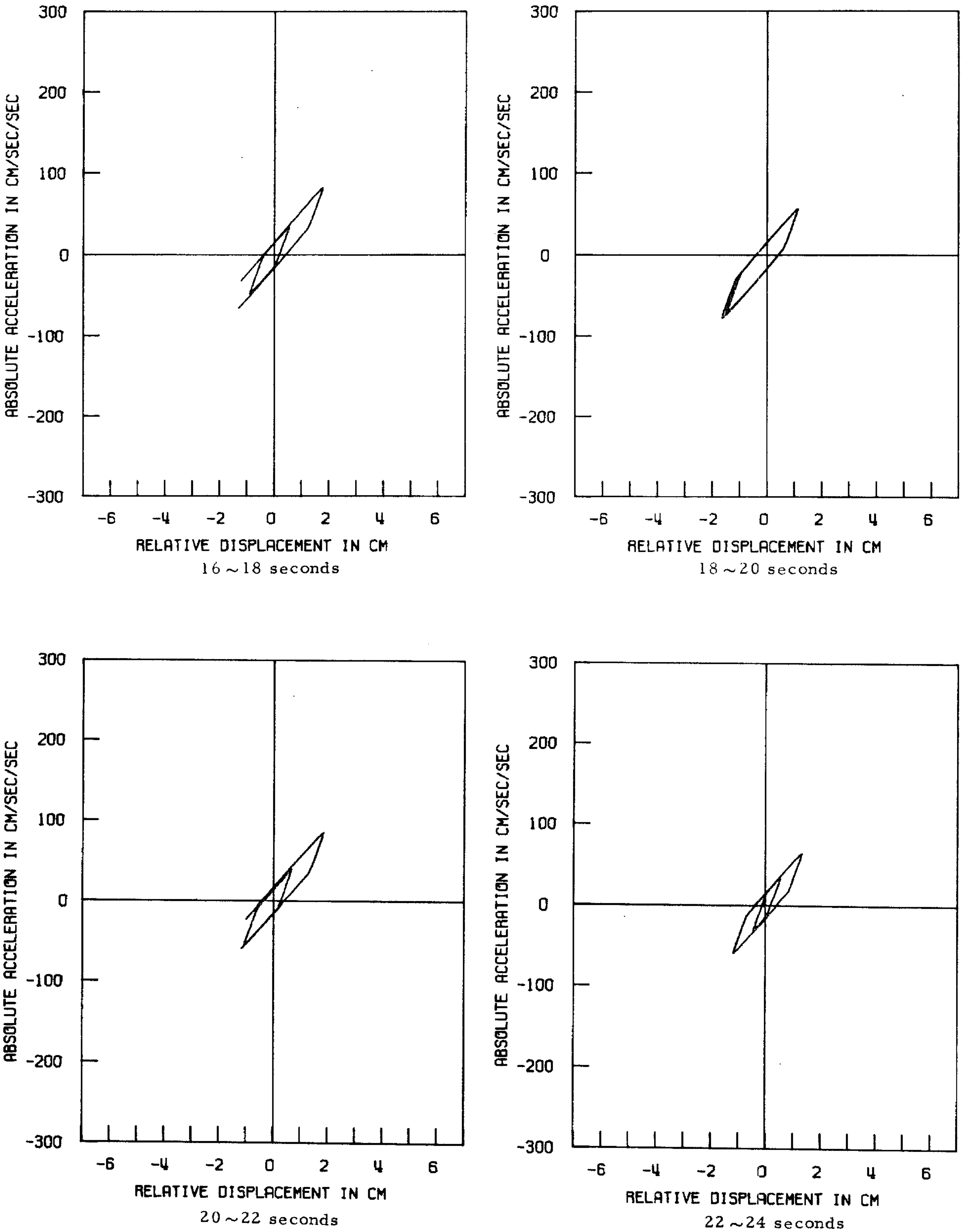
Hysteretic response of stationary, bilinear hysteretic model of fundamental mode.



HYSTERETIC RESPONSE OF STATIONARY BILINEAR MODEL

FIGURE 14b

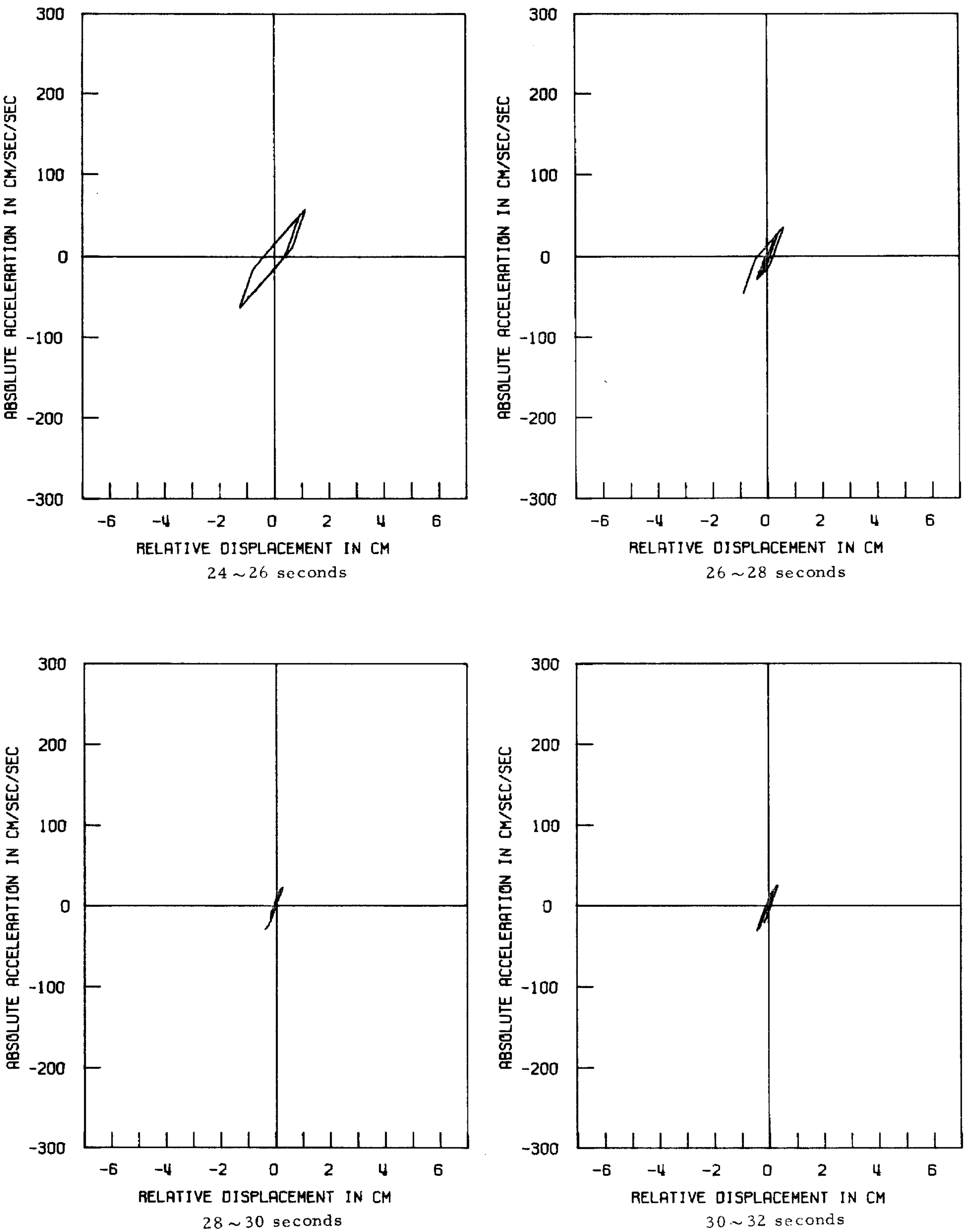
Hysteretic response of stationary, bilinear hysteretic model of fundamental mode. (Cont'd.)



HYSTERETIC RESPONSE OF STATIONARY BILINEAR MODEL

FIGURE 14c

Hysteretic response of stationary, bilinear hysteretic model of fundamental mode. (Cont'd.)



HYSTERETIC RESPONSE OF STATIONARY BILINEAR MODEL

FIGURE 14d

Hysteretic response of stationary, bilinear hysteretic model of fundamental mode. (Cont'd.)

accuracy of the equivalent linear parameters given in Figures 7 and 10 and also to investigate the nonstationary characteristics of the response. The time-dependent equivalent natural frequency ω_{eq} and damping factor h_{eq} for the nonstationary model were selected from examination of the data, and are shown in Figure 15.

During the computation of the response, the changes of stiffness were implemented at times when the relative displacement was zero so as not to cause any permanent deformation. The calculated response of this oscillator, shown in Figure 16, agrees quite well with the response of the first mode shown in Figure 8. Thus, very good agreement with the observed behavior can be obtained by considering the structure to behave like a linear oscillator whose natural frequency and damping factor change during the course of the earthquake response. The good agreement suggests also that the analysis presented above can give sufficiently accurate nonstationary equivalent linear parameters. It is seen from Figure 15 that the stiffness of the equivalent linear system degrades to a constant value, whereas the equivalent damping factor of the system first increases and then decreases to a value somewhat lower than the peak response, but higher than the initial value.

Nonstationary, Bilinear Model

In this section, a nonstationary, deteriorating model of bilinear hysteresis is proposed to describe the response of the fundamental mode. The model chosen consists of four different bilinear hysteretic relations all having the same second slope. The time-dependent characteristics of the stiffness and energy dissipation capacity of the library are represented by changing the stiffness of the first slope and the yielding displacement.

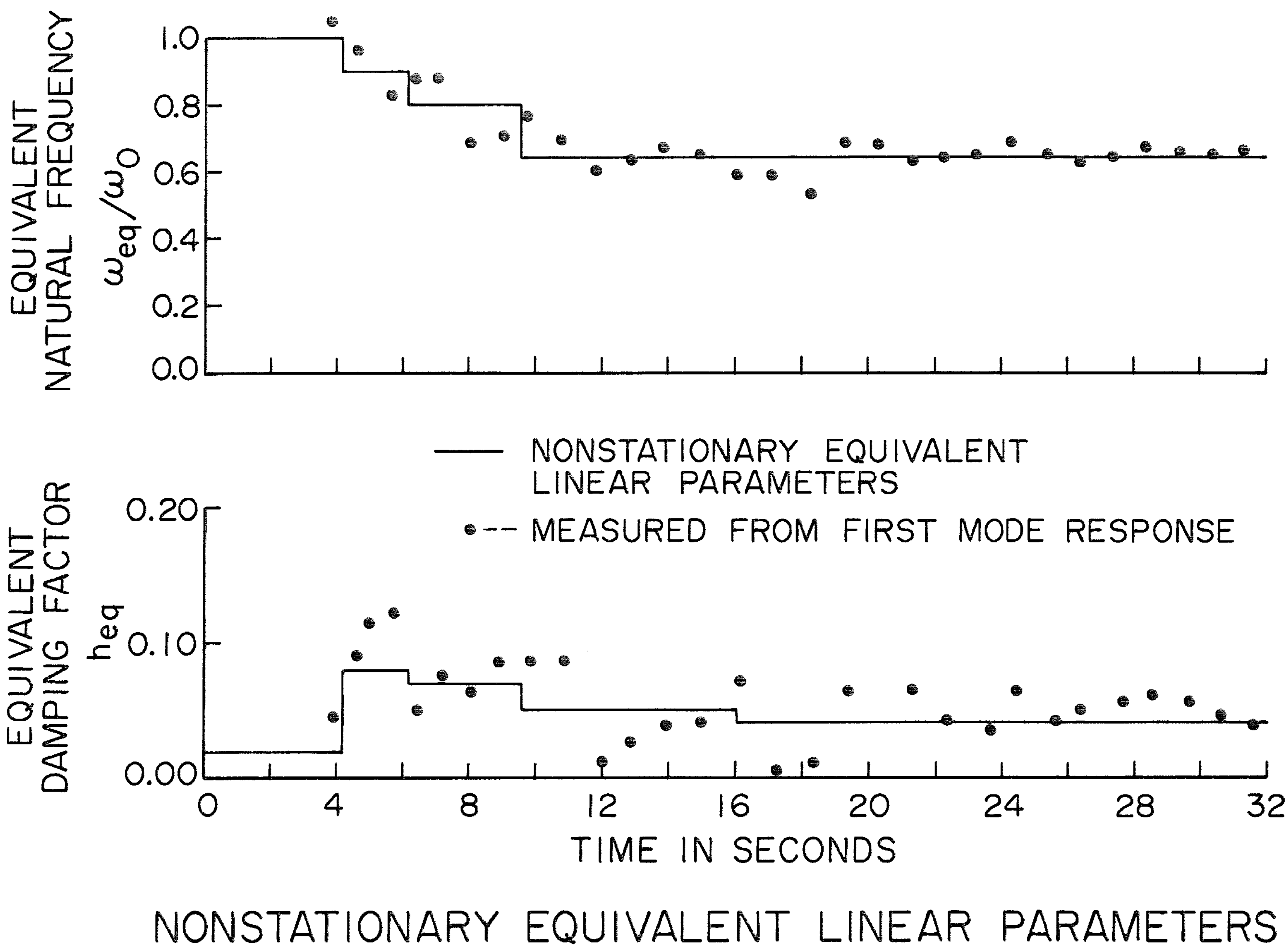
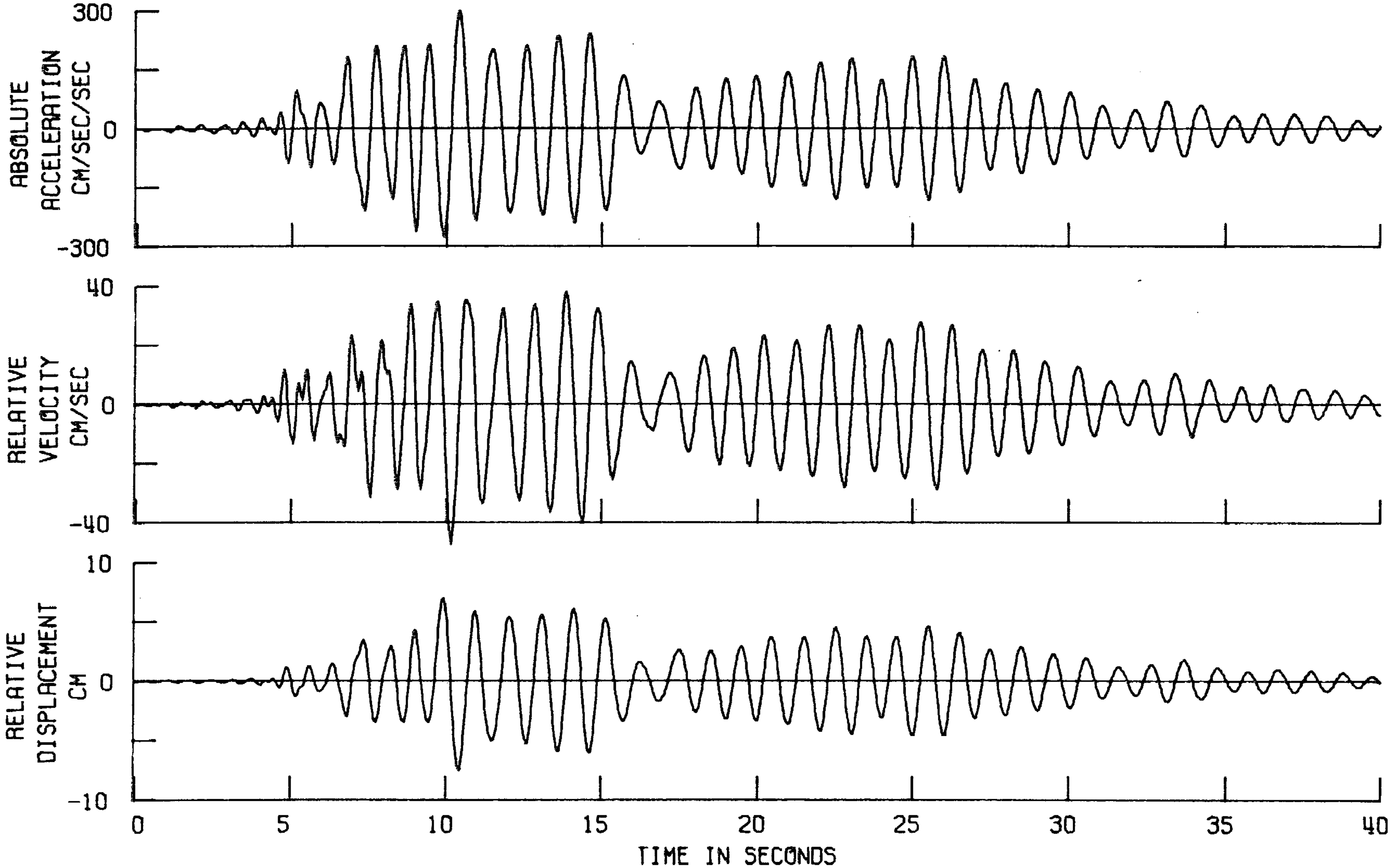


FIGURE 15

Equivalent damping factors and natural frequencies for nonstationary, equivalent linear model.



RESPONSE OF NONSTATIONARY EQUIVALENT LINEAR MODEL

FIGURE 16

Absolute acceleration, relative velocity and relative acceleration of nonstationary linear model.

Guided by the results of previous analyses, the yielding displacement for the bilinear model was taken as large as 1.0 cm during the initial portion of the response; and for the latter portion of the response, a smaller value of 0.07 cm was used. The four different bilinear relations employed to model the nonstationary characteristics of the structure are consistent with the equivalent linear parameters shown in Figure 15, and hysteresis loops for these relations are shown in Figure 17. The loss of stiffness with time and the decreasing capacity for dissipating energy are apparent from Figure 17.

During the computations of response the transition from one hysteretic model to another was controlled to avoid jumps in the restoring force. The calculated values of absolute acceleration, relative velocity and relative displacement for the nonstationary bilinear hysteretic model are plotted in Figure 18. Figure 19 shows the calculated hysteretic behavior of the model, and is to be compared with Figure 9. Comparing the responses in Figure 18 with those of the first mode in Figure 8, it is seen that the two results agree very well except for a few peaks around 8 secs. Comparing the hysteretic diagrams in Figures 9 and 19, the calculated hysteretic behavior produced by the nonstationary bilinear model seems to represent the deteriorating characteristics of the restoring force of the structure fairly well. The agreement might be improved by the introduction of another, fifth model, or by changing the properties of the four used, but the main features of the hysteretic characteristics seem to be represented reasonably well by the nonstationary model used in the analysis.

CONCLUSIONS

Of the four simple models used to describe the E-W response of the fundamental mode of the library during the San Fernando earthquake, the best agreement was achieved by the use of the two nonstationary

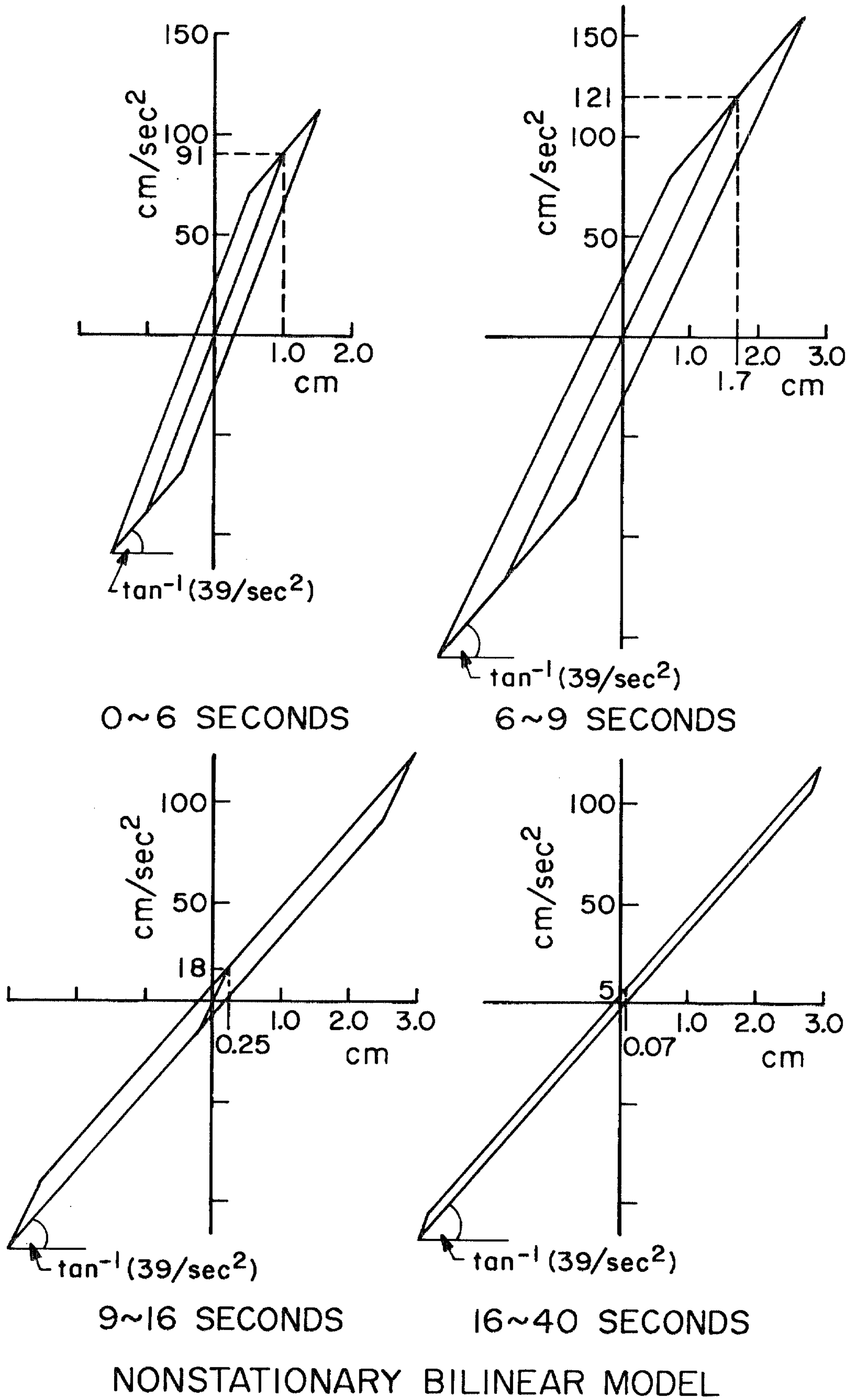
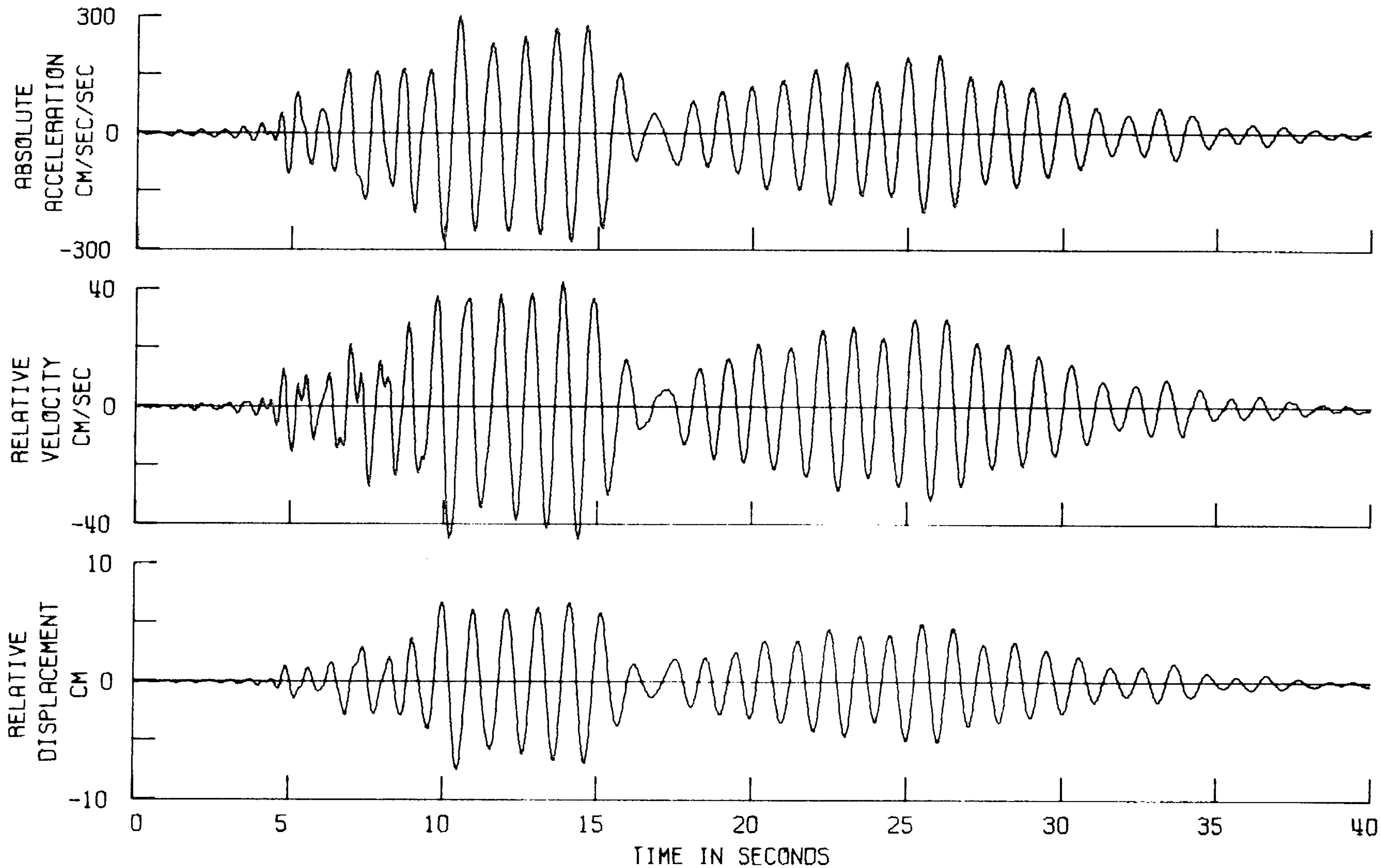


FIGURE 17

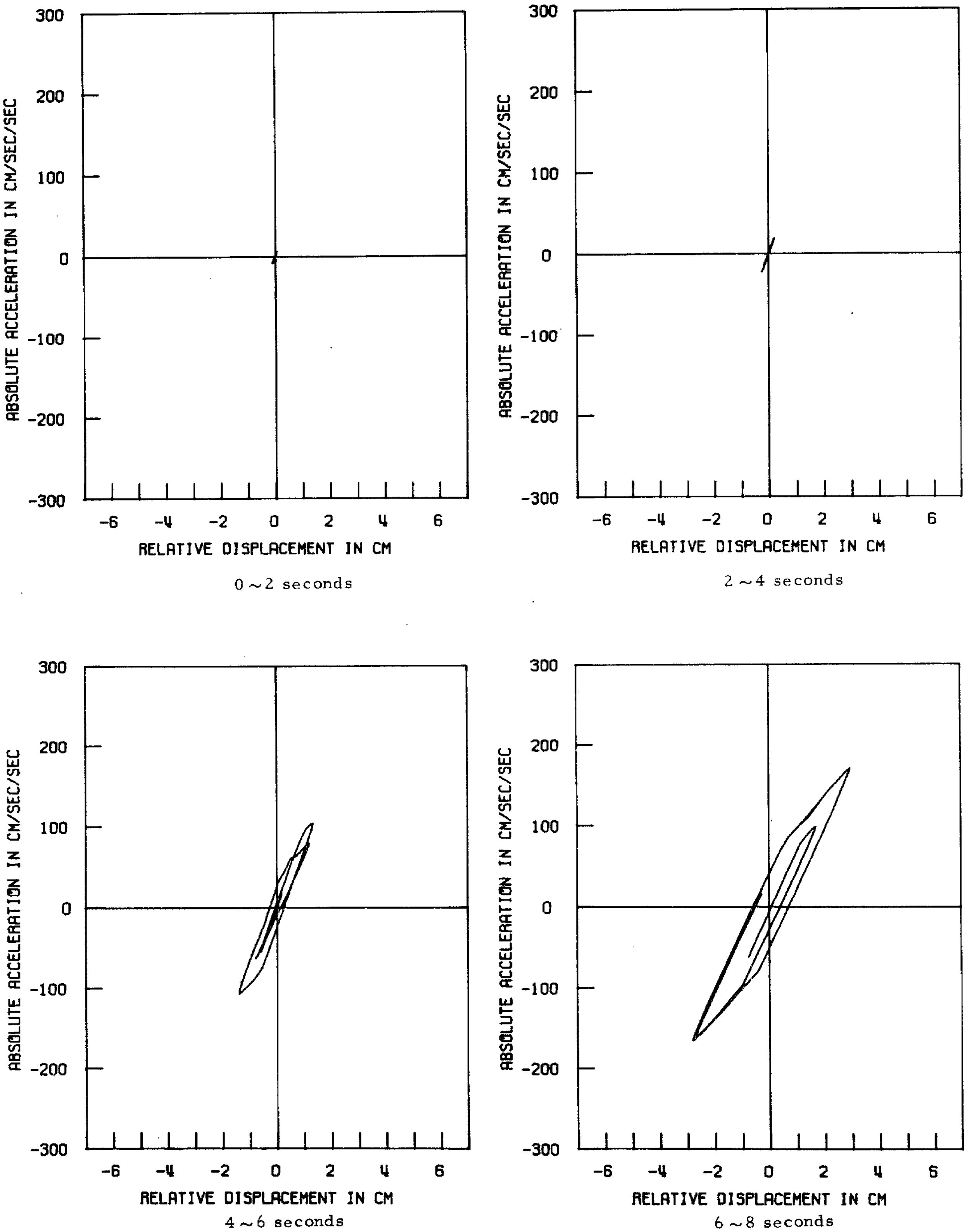
Typical hysteresis loops for the nonstationary, bilinear hysteretic model.



RESPONSE OF NONSTATIONARY BILINEAR MODEL

FIGURE 18

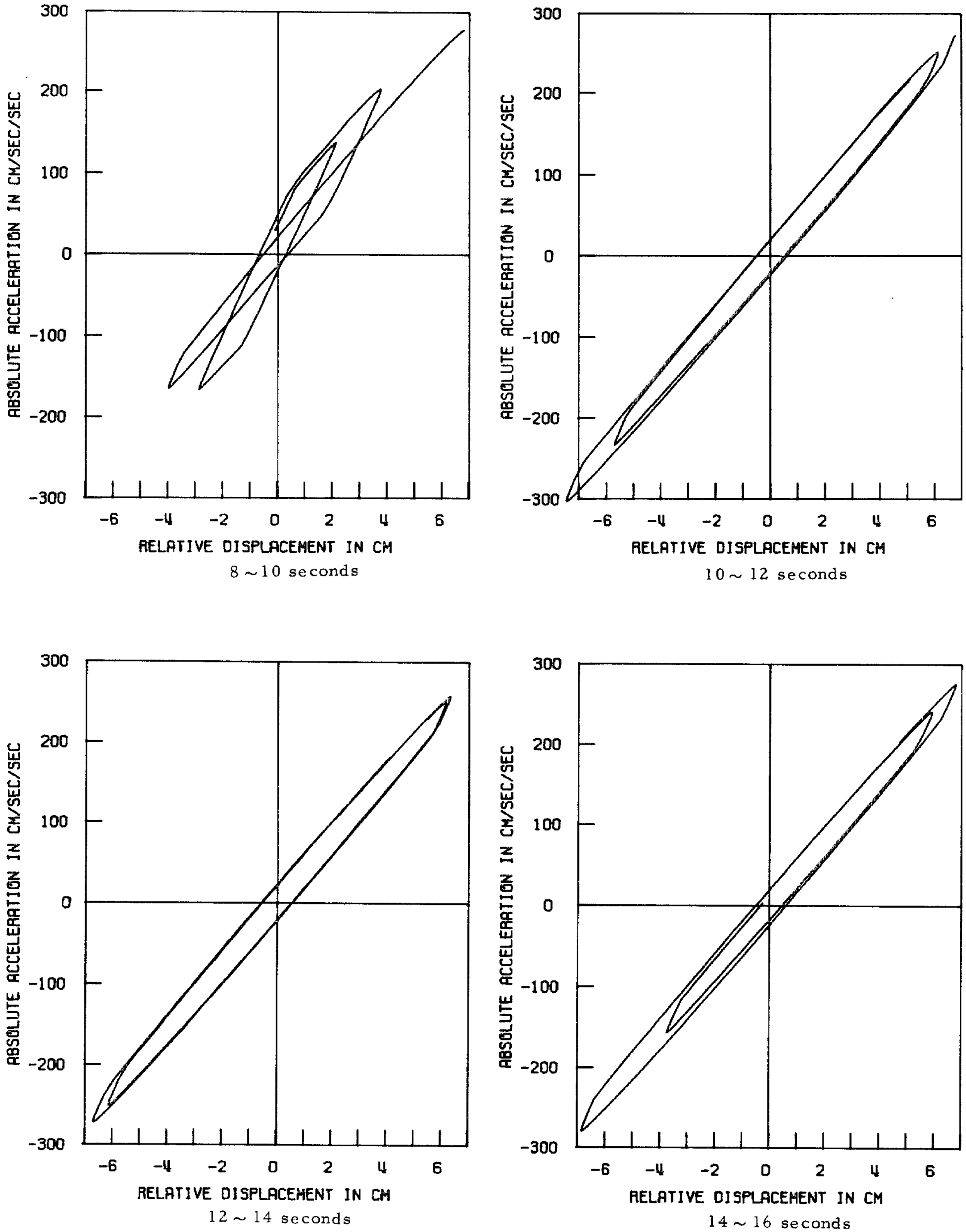
Absolute acceleration, relative velocity and relative displacement of the nonstationary, bilinear hysteretic model.



HYSTERETIC RESPONSE OF NONSTATIONARY BILINEAR MODEL

FIGURE 19a

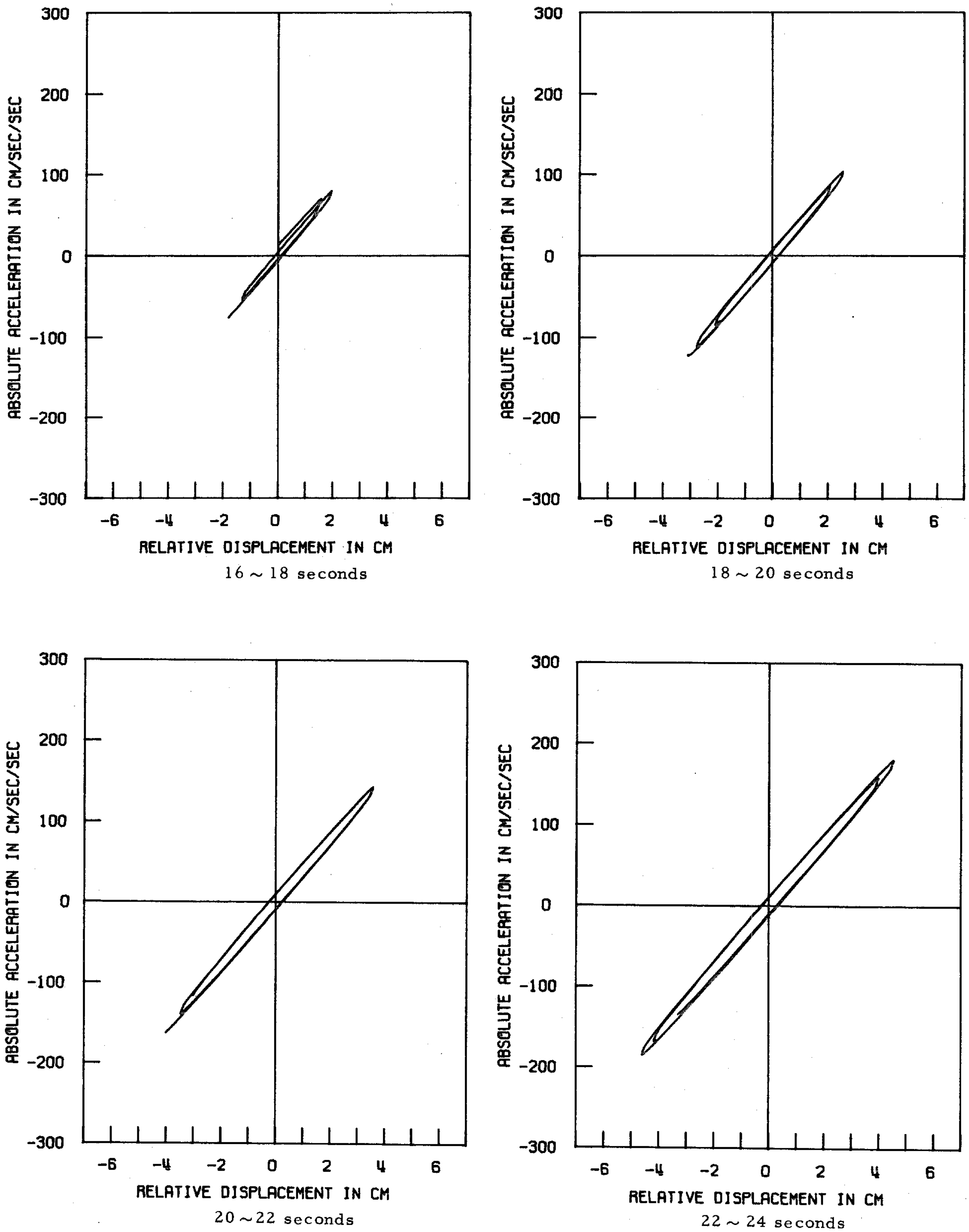
Hysteretic response of nonstationary, bilinear hysteretic model of fundamental mode.



HYSTERETIC RESPONSE OF NONSTATIONARY BILINEAR MODEL

FIGURE 19b

Hysteretic response of nonstationary, bilinear hysteretic model of fundamental mode. (Cont'd.)



HYSTERETIC RESPONSE OF NONSTATIONARY BILINEAR MODEL

FIGURE 19c

Hysteretic response of nonstationary, bilinear hysteretic model of fundamental mode. (Cont'd.)

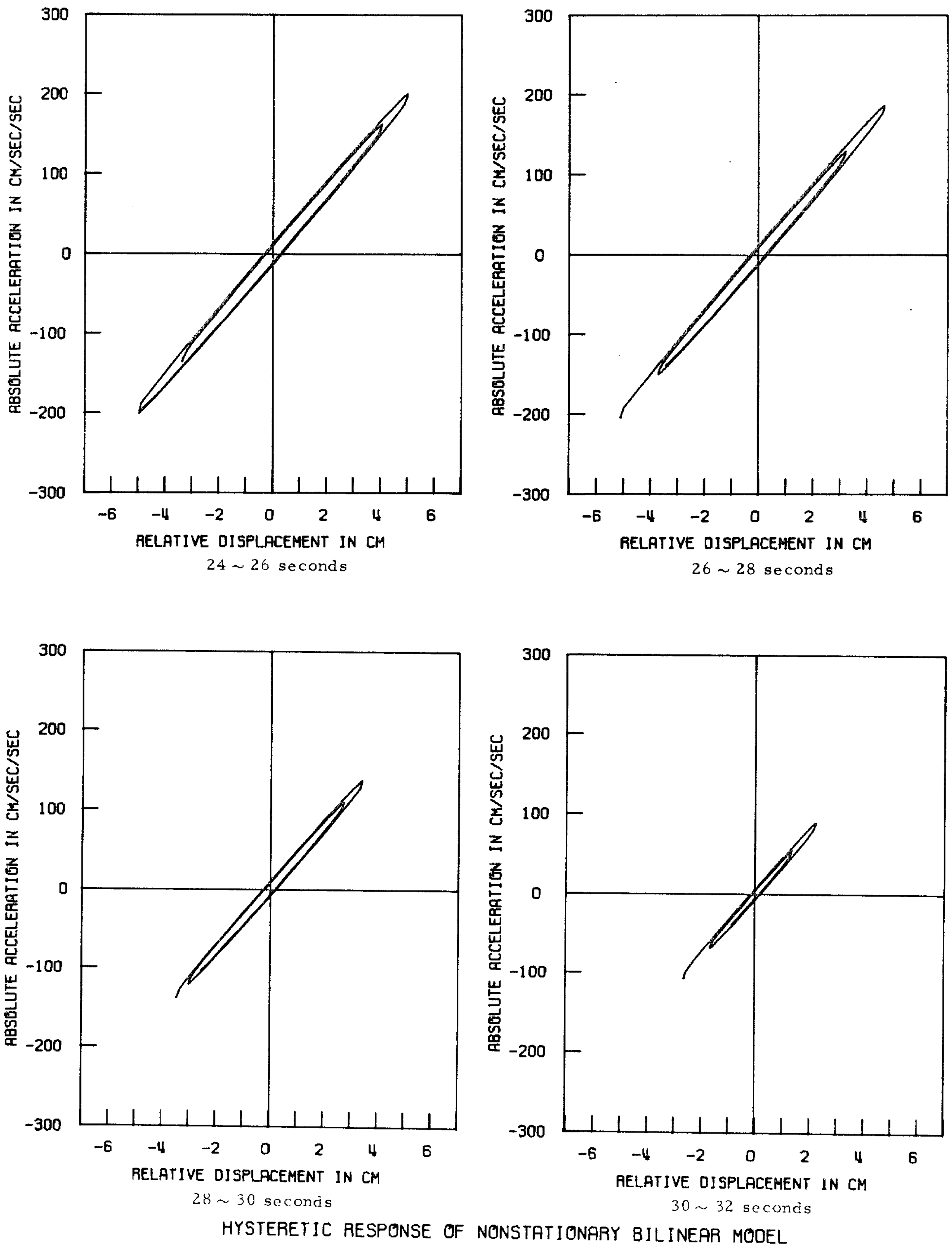


FIGURE 19d

Hysteretic response of nonstationary, bilinear hysteretic model of fundamental mode (Cont'd.)

oscillators. The two stationary models, an equivalent linear model and a bilinear hysteretic model, also with constant properties, were not capable of duplicating the earthquake response nearly so well as the nonstationary oscillators. The simpler, stationary models did give maximum responses close to that observed in the earthquake, however, so that their use would have produced valid information in an analysis intended for design.

The two nonstationary models that gave good agreement were an equivalent linear model with properties that were changed at four times during the earthquake, and a bilinear hysteretic model that also changed properties four times during the response. Equally good agreement was obtained with either model, and it is concluded that any of the more common hysteretic models giving the general trend of equivalent natural frequency and equivalent damping factor shown in Figure 15 probably could be made to give good agreement between observed and calculated responses. In doing any such analyses, however, it does appear necessary to change the properties of the model during the course of the response; it seems doubtful that any of the simple, non-degrading hysteretic models could be capable of giving the degree of agreement shown by the nonstationary models.

The results of the analysis, and study of the observed E-W response of the library, clearly indicate a significant decrease in the stiffness and energy dissipation capability of the building during the course of the earthquake response. This is perhaps most easily seen in Figure 15. It is not possible to relate the changes, with confidence, to any observed damage to the building, nor is it possible to ascertain whether the changes were sudden or gradual. It seems quite possible, however, that the observed behavior is at least partly a consequence of the behavior of the precast concrete panels that contain the windows, and it seems to the authors that relatively rapid or sudden changes in properties are more likely to have occurred than gradual ones.

The simultaneous measurement of the ninth floor and basement motions allowed the calculation of the relative response which, in this case, could be used to construct an experimental estimate of the hysteretic response of an oscillator modelling the fundamental mode of the structure. To this extent it was possible to study the actual hysteretic behavior of the library and thereby to judge the type of hysteresis that best described the response. The method used in this report appears to be a promising one for studying earthquake response of hysteretic structures, even though some difficulties exist in obtaining hysteretic trajectories from the response. To study the hysteretic behavior in more detail, in particular to determine where in the structure the hysteresis might be concentrated, would require more instrumentation than is present in the library. It is concluded that one instrument per floor, all with a common timing signal, would be the minimum required to give the information needed.

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APPENDIX

This appendix describes the calculation of the participation factor of the fundamental mode and the weighting factor for the first mode for response as measured on the roof. These factors are required to scale the measured response on the roof to the response of the single-degree-of-freedom oscillator that models the fundamental mode of the building.

The equation of motion for earthquake response of an n degree-of-freedom system such as the library can be written as

$$M\{\ddot{x}\} + C\{\dot{x}\} + K\{x\} = -M\{1\}\ddot{z}(t) \tag{A1}$$

in which M, C and K are the n x n mass, damping and stiffness matrices, respectively. The vector $\{x\}$, $(\{x\}^T = \{x_1, x_2, \ldots, x_n\})$ denotes the relative displacements, $\{1\}$ symbolizes the vector $\{1\}^T = \{1, 1, \ldots, 1\}$, and $\ddot{z}(t)$ is the acceleration of the base of the structure.

The matrix of mode shapes Φ is defined by

$$\Phi = [\{\phi_1\}, \{\phi_2\}, \ldots, \{\phi_n\}] \tag{A2}$$

in which the column vectors are the individual mode shapes, i. e., $\{\phi_i\}^T = (\phi_{1i}, \phi_{2i}, \ldots, \phi_{ni})$ defines the i^{th} mode.

Letting

$$\{x\} = \Phi\{\xi\} \tag{A3}$$

substituting into Eq. A1, and multiplying by Φ^T gives

$$\Phi^T M \Phi\{\ddot{\xi}\} + \Phi^T C \Phi\{\dot{\xi}\} + \Phi^T K \Phi\{\xi\} = -\Phi^T M\{1\}\ddot{z}(t) \tag{A4}$$

Under the assumption that the damping matrix C can be diagonalized by the same transformation Φ which diagonalizes M and K, the individual equation in matrix equation A4 will be uncoupled. A typical equation will have the form

$$\ddot{\xi}_i + \frac{\{\phi_i\}^T C\{\phi_i\}}{\{\phi_i\}^T M\{\phi_i\}}\ \dot{\xi}_i + \frac{\{\phi_i\}^T K\{\phi_i\}}{\{\phi_i\}^T M\{\phi_i\}}\ \xi_i = -\frac{\{\phi_i\}^T M\{1\}}{\{\phi_i\}^T M\{\phi_i\}}\ \ddot{z}(t) \qquad \text{(A5)}$$

The coefficient of $\ddot{z}(t)$ is the participation factor, a_i, for the i^{th} mode, and in particular

$$a_1 = \frac{\{\phi_1\}^T M\{1\}}{\{\phi_1\}^T M\{\phi_1\}} \qquad \text{(A6)}$$

Assuming that the solutions to equation A5 are known, the response of each of the n masses can be found by use of equation A3. If an index of 1 in a mode corresponds to the roof, the roof displacement is

$$x_1 = \phi_{11}\,\xi_1 + \phi_{12}\,\xi_2 + \ldots + \phi_{1n}\,\xi_n \qquad \text{(A7)}$$

The modal ordinate ϕ_{11} is herein called the weighting factor.

From the test data (10), the first mode of the nine-story library was found as

$$\{\phi_1\}^T = \{1.00, 0.87, 0.74, 0.62, 0.51, 0.40, 0.28, 0.20, 0.11, 0.04\} \quad \text{(A8)}$$

where values for intermediate floors have been interpolated from the measurements. The mass matrix is given in the same reference as

$$M = \frac{2600\ \text{kips}}{g}\begin{bmatrix} 1.0 & & & & & & & & & \\ & 0.75 & & & & & & & & \\ & & 0.75 & & & & 0 & & & \\ & & & 0.75 & & & & & & \\ & & & & 0.75 & & & & & \\ & & & & & 0.75 & & & & \\ & & 0 & & & & 0.75 & & & \\ & & & & & & & 0.75 & & \\ & & & & & & & & 0.94 & \\ & & & & & & & & & 0.88 \end{bmatrix} \qquad \text{(A9)}$$

Evaluating equation A6 it is found that

$$a_1 = 1.44 \tag{A10}$$

and from equation A8

$$\phi_{11} = 1.00 \tag{A11}$$

The product of these two factors, 1.44, is the desired ratio, i.e., the response of an oscillator subjected to the recorded base acceleration should be multiplied by 1.44 before comparison with the fundamental mode response, as measured on the roof. For the calculations in this report, the rounded value of 1.4 has been used.